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FREQUENTIST APPROACH FOR PARAMETRIC-SURVIVAL MODELS WITH APPLICATION TO RIGHT CENSORED LIVER CIRRHOSIS DATA

[1]Abubakar Muhammad Auwal, [1]Waheed Babatunde Yahya, [1]Daniel Victoria and [1]Ahmed Ibrahim

Article Info

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Abstract

The aim of this study was to fit an appropriate parametric survival model to right-censored liver cirrhosis data using the frequentist approach. Secondary data obtained from selected hospital facilities were used in this study. The collected data were analyzed using survival analysis. The Global test results confirmed that the constant hazard assumption was met. The results of the model comparison using the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) revealed that the Weibull proportional hazard (PH) model and the lognormal accelerated failure time (AFT) model outperformed the other models considered in this study. Overall, the lognormal AFT model outperformed the Weibull PH model. Based on this model, it was discovered that drugs and liver status were significant predictors of survival in patients with liver cirrhosis. Based on these findings, it was recommended that patients with liver cirrhosis who were on drugs should adhere strictly to their medication and also consider regular liver function tests to ensure that their liver is in good state.

INTRODUCTION

The liver is the largest gland that performs various vital functions in the body and plays a major role in maintaining health by changing absorbed food and eliminating toxins. It is responsible for a variety of functions, including regulating glucose metabolism, producing and secreting bile for digestion and absorption of fats from the digestive system, producing proteins required by blood plasma, and removing metabolic waste from the blood and secreting it into the bile. Liver cirrhosis is a serious and progressive disorder characterized by the liver's response to lesions occurring in it, where some fibrosis appears in the liver due to chronic liver diseases. Liver fibrosis is an overgrowth of collagen that causes a change in the shape of the liver in patients with cirrhosis. These changes lead to pressure on the blood vessels of the liver and its small pores (Bataller & Brenner, 2019).

Liver cirrhosis is characterized by irreversible damage, in response to which liver tissue is replaced by fibrosis and scarring (Zhou et al, 2014). If unchecked, cirrhosis may progress to end-stage liver disease and cause death.

[1] Department of Statistics, Nasarawa State University

This may or may not be preceded by hepatocellular carcinoma, a type of liver cancer (Llovet, et al., 2016). Globally, cirrhosis caused approximately 1.32 million deaths, representing 2.4% of all deaths in 2017. At that time, there were approximately 112 million compensated and over 10.6 million decompensated cirrhosis cases worldwide, with a prevalence of approximately 1.4% (Sepanlou et al., 2020). In comparison, there were 4.5 million adults diagnosed with liver disease in the United States in 2018, with cirrhosis of the liver being responsible for 56,585 deaths and ranked 9th amongst all causes of death in 2021 (Hawa, 2023). The prevalence of cirrhosis in the US is estimated to be approximately 0.27%.5 Although some common causes are infectious hepatitis due to Hepatitis B and C viruses, alcohol-related liver disease, and nonalcoholic fatty liver disease, the predominant etiology varies according to the geographic location. Although infectious hepatitis remains the most common cause of liver cirrhosis globally, alcohol-related liver disease dominates countries like India, Australia, Russia, and the continents of Europe and South America (Huang, et al., 2023).

In advanced cases, the only definitive treatment for liver cirrhosis is liver transplantation (LT) (Grattagliano, et al. 2011). The two major problems that may occur in a patient after LT are liver rejection and infection. Immunosuppressive drugs are prescribed to prevent liver transplant rejection. These drugs slow down or stop the immune system to prevent the rejection of the new liver. Furthermore, patients are at high risk of infections (Zahmatkeshan et al., 2017). Thus, identifying factors affecting the survival of patients with liver cirrhosis is of paramount importance.

There is growing interest in applying statistical methods in medicine. One area of statistics that has received much attention in medicine is the survival analysis method. Survival analysis is the analysis of time-to-event data (Isaac, 2019). Some typical examples of time-to-event data in medical science include time to death or time until infection (Lawless, 2011).

The approaches used in survival analysis are different from those used in other statistical techniques for the following reasons: time is always the response variable; staggered entries (units in the study may have different entrance times) may occur, but this has no bearing on the survival analysis method since it considers the duration of observation rather than the time of entry; and finally, normalcy assumptions are violated because survival data are typically skewed (Isaac, 2019).

Parametric hazards models, such as exponential, Weibull, Gompertz; parametric survival models such as Lognormal, Log-Logistic, Gamma, e.t.c. have been generally used to analyze time-to-event data. The methods give similar results, but each method is unique, usually under specific or no assumptions (Shankar *et al*. 2019).

The parametric proportional Hazard (PH) and acceleration failure time (AFT) models are the two most popular and frequently used parametric hazard-based regression models. The proportional hazard models are based on the constant hazard assumption (constant hazard) while the AFT is the alternative model when the proportional hazard assumption is not met. (Khanal et al., 2019).

In this paper, we present the following points:

1. Basic Functions of Survival Data Analyses

There are four basic functions that play pivotal roles in modeling survival data. These include the probability density function, survivor function, hazard function, and cumulative hazard function. This section briefly discusses these functions.

a) Survivor Function

The survivor function denoted by $s(t)$ is defined as the probability that an individual survives at least t units. mathematically, survival function as given below:

$$S(t) = p(T \geq t) \qquad (2.3)$$

The survival function denoted is a monotone, non-increasing, and left continuous function, with $S(t) = 1$ and $lim_{t\to\infty} S(t) = 0$. Survival curve is the graphical representation of the survival function $s(t)$ as a function of t. The survival function is one minus the Cumulative Density Function (CDF) of T, that is $S(t) = 1 - F(t)$, where $F(t)$ is the cumulative density function of T.

b) Probability density function:

Mathematically, the probability density function (pdf) denoted by $f(t)$ is defined by

$$f(t) = -\frac{d}{dt}S(t) = lim_{\Delta t \to 0}\frac{\Pr(t \le T < t+\Delta t}{\Delta t} \quad (2.4)$$

where indicates the rate of increase of $1 - S(t)$, so that

$$s(t) = \int_t^{\infty} f(s)ds \quad (2.5)$$

c) Hazard Function

The hazard function which is denoted by $h(t)$, is defined as

$$h(t) = lim_{\Delta t \to 0}\frac{\Pr(t \le T < t+\Delta t | T \ge t}{\Delta t} = \frac{f(t)}{S(t)} \quad (2.6)$$

Equation 2.2 gives the instantaneous rate per unit time for an event to occur, given that the individual survives up to time *t*. Note that $h(t)\Delta t$ is the approximate probability of the event to occur in $[t, t + \Delta t)$, given survival up to time *t*. The hazard function is also known as the hazard rate or failure rate.

According to Lawless (2003), it is easy to verify that:

$$h(t) = \frac{S'(t)}{S(t)} = -\frac{d}{dt}logS(t) \quad (2.7)$$

so that

$$logS(t) = -\int_0^t h(s)ds \quad (2.8)$$

From equation 2.8, the survival function can be written after taking the exponent as follows:

$$S(t) = exp\left\{-\int_0^t h(s)ds\right\} \quad (2.9)$$

From 2.6 and 2.9. it follows that

$$f(t) = h(t)S(t) = h(t)\, e^{\left\{-\int_0^t h(s)ds\right\}} \quad (2.10)$$

d) Cumulative Hazard Function

The cumulative hazard function, denoted by H(t), is defined as follows:

$$H(t) = \int_0^t h(s)ds \quad (2.11)$$

According to Collett (2003), equation 2.11 can be interpreted as the expected number of events that occur up to a given time, namely, *t*. The relationship $H(t) = -logS(t)$ plays an important role in determining the adequacy of a parametric survival model and in formulating likelihood functions for censored survival data.

2. Regression Models for Time-to-Event Data

There are two popular approaches for the regression analysis of survival data: Accelerated Failure Time and Proportional Hazards models. In the following, we describe the AFT and PH models assuming $\Psi(Z\beta) = \exp(Z'\beta)$.

i) The Accelerated Failure Time Model

In the AFT model, covariates act multiplicatively on survival time. Let T be a random sample of survival times and Z is a vector of covariates such that $Z = Z_1, \dots, Z_P$, the AFT model defines the relationship of the survival function for every time $t \in T, S(t|Z)$, and the covariates are as follows:

$$S(t|Z) = S_0\left(te^{Z\beta^t}\right) \quad (2.12)$$

where $S_0(.)$ is the baseline survival function, Z is the vector of covariates, β is the regression coefficient vector. The factor e^Z in equation 2.12 is known as the accelerated factor, which accelerates or decelerates the survival function with covariate $Z = 0$. The AFT model assumes that the effects of the covariate are fixed and multiplicative by the accelerated factor on the time scale of t.

The linear relationship between the covariates and survival time can also be illustrated using the natural logarithm of survival time and the covariate Z, as follows

$$lnT = \mu + \varphi^t Z + \sigma V \quad (2.13)$$

where μ is the slope, $\sigma > 0$ is an unknown scale parameter, $\varphi^t = (\varphi_1, \dots, \varphi_p)$ is a vector of regression coefficients, $\varphi = -\beta$, and V is a distribution error which is a random variable and assumed to follow a certain parametric distribution. For each distribution of V, there is a related parametric for T. The AFT model is named after the

distribution of T rather than the parametric distribution of $ln\,T$. Some popular parametric distributions, which correspond to the AFT model, are Weibull, exponential, log-normal, gamma, and log-logistic.
The survival function of $T_i, i = 1, \dots, n$ can be expressed as follows:
$S_i(t) = P(T_i \geq t) = P(lnT_i \geq lnt) = P(\mu + \varphi^t Z + \sigma V \geq lnt)$
$P(V_i \geq \frac{lnt-(\mu+\varphi^t Z)}{\sigma})$ (2.14)
As stated earlier, the error term V in (2.13) is assumed to follow a standard parametric distribution, such as the normal, extreme, or logistic. These results lead to log-normal, Weibull, and log-logistic models for *T*, respectively.

ii) Proportional Hazard Model

the proportional hazard assumption indicates that the effect of a covariate is to increase or decrease the hazard by a proportionate amount that does not depend on *t*. Under this assumption, the hazard function with covariate vector m (fixed/time dependent) is given follows:
$h(t; Z) = h_o(t)\exp(Z'\beta)$ (2.15)
where $h_o(t)$ is the baseline hazard. Thus, the hazard ratio comparing any two specifications of the covariates, say m and Z^*, is
$\frac{h(t;Z)}{h(t;Z^*)} = exp(Z' - Z^{*\prime})\beta$ (2.16)
which is constant over time. Therefore, the hazard for one individual is proportional to the
Hazard to any other individual, where the proportionality constant is not dependent on time. The
the survivor and probability density functions in the PH model are
$s(t; Z) = [S_0(t)]^{\exp(Z'\beta)}$ (2.17)
$f(t; Z) = f_0(t)\exp(Z'\beta)[S_0(t)]^{\exp(Z'\beta)}$ (2.18)
A proportional-hazard model developed by assuming an arbitrary and unspecified baseline hazard function $h_o(t)$ in equation 2.16 leads to the popular Cox proportional-hazard (CPH) model (Cox 1972). The CPH mode does not rely on the distributional assumption of the outcome variable. However, a fully parametric proportional hazard model can be expressed by assuming a baseline distribution for $h_o(t)$. The classical exponential and Weibull distributions can be used for this purpose. For instance, suppose that *T* has a Weibull distribution with parameters k and φ. From equation 2.16, the Weibull PH model can be written as
$h(t; Z) = h_o(t)exp(Z'\beta) = k\varphi^k t^{k-1}\exp(Z'\beta) = k(\varphi\exp(\frac{Z'\beta}{k}))^k t^{k-1}$ (2.19)
Equation 2.18 again gives the Weibull hazard with $\varphi^* = \varphi e^{\frac{Z\beta}{k}}$. Therefore, we conclude that the Weibull is closed under the PH relationship. In fact, the Weibull model is the only family that is closed under both the multiplication of the failure time (AFT framework) and the multiplication of the hazard function (PH framework) by an arbitrary nonzero constant (Kalbfleisch & Prentice, 2002). For the Weibull AFT model, $\varphi^* = \varphi e^{-Z\beta}$, therefore the regression coefficients of the AFT and PH models with the Weibull baseline distribution are related as follows: β for the PH model with the Weibull baseline distribution is equal $-k\beta$ for the Weibull AFT model (Khan, 2019).
The likelihood function of the parametric PH model can be written using equation 2.38 as follows:
$L(\vartheta) = \sum_{i=1}^{n} \delta_i[\log h_0(t_i, \omega) + Z'\beta] - \sum_{i=1}^{n} H_0(t_i, \omega)\exp(Z'\beta)$ (2.20)
Where $\vartheta = t_i, \omega,\ \omega$ is the vector of parameters of the baseline distribution and β is the vector of regression coefficients. Equation 2.20 can be maximize directly using Newton-Raphson optimization algorithm.
Cox (1972) proposed partial-likelihood methods to estimate β for Cox PH model, with the baseline hazard $h_0(t)$ left unspecified.
Suppose that we have n individuals under study. Let assume $t_1, t_2, t_3, \dots, t_n$ denote their failure/censoring times and $\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n$ denotes censoring indicators. The observed data for individual *i* consist of $\{t_i, \alpha_i, Z_i\}, i = 1,2,3, \dots, n$.
Let $t_{(1)} < t_{(2)} <, \dots, t_{(k)}$ denote the unique-order observed failure times, $i_{(k)}$denotes the individual with the failure time $t_{(k)}, k = 1, 2, \dots, k$ and $R(t) = \{i: t_i \geq t)$ denotes the set of all individuals at risk for failure at time

t, popularly known as the risk set. Using equation 2.16, the conditional probability that the individual $i_{(k)}$ failed at time $t_{(k)}$ given one subject at risk failed at that time is given as follows:

$$\frac{h_0(t_{(k)})\exp(Z'_{i(k)}\beta)}{\sum_{l\epsilon R(t_{(k)})} h_0(t_{(k)})\exp(Z'_l\beta)} = \frac{\exp(Z'_{i(k)}\beta)}{\sum_{l\epsilon R(t_{(k)})}\exp(Z'_l\beta)} \quad (2.21)$$

After considering the product of such term over all *k = 1, 2…, k,* equation 2.48 will result to

$$L(\beta) = \prod_{k=1}^{k} \frac{e^{Z'_{i(k)}\beta}}{\sum_{l\epsilon R(t_{(k)})} e^{Z'_l\beta}} \quad (2.22)$$

Equation 2.22 is not a usual likelihood function because it is the product of the conditional probabilities, where the conditioning event changes over time. Cox (1972) argued that this
should behave roughly like a likelihood function and can be used as a basis for inference of the following: β. This function is called the partial likelihood function in the Cox PH model. Note that (2.22) can also be written as

$$L(\beta) = \prod_{i=1}^{n} \left[\frac{e^{Z'_i\beta}}{\sum_{l\epsilon R(t_{(k)})} e^{Z'_l\beta}} \right]^{\delta_i} \quad (2.23)$$

The partial log-likelihood is given as

$$\ell(\beta) = \sum_{i=1}^{n} \delta_i [\, Z'_i\beta - \log(\sum_{l\epsilon R(t_{(k)})} e^{Z'_l\beta})] \quad (2.24)$$

Thus, the Newton-Raphson iteration or any other optimization method can be used to obtain the maximum likelihood of β from equation 2.24.

3. Summary of Literature Review and Gaps

This chapter presents the literature review, starting with a conceptual, empirical, and theoretical review. Based on the empirical literature reviewed, it was discovered that most studies conducted on survival analysis focused on nonparametric and semiparametric survival analysis methods, and only a few considered the parametric method. In addition, few empirical studies have examined the factors affecting the survival of patients with liver cirrhosis. Thus, this study seeks to fill these gaps identified in the literature.

4. Data Presentation

The data used are presented in appendix A. The data were also presented using descriptive statistics (Table 4.1.

Table 4.1: Descriptive Statistics Results

Variable	Category	Frequency	Percentage
Hepatitis Type	Hepatitis Type A	69	23.7
	Hepatitis B	141	48.5
	Hepatitis C	81	27.8
Educational Level	No Education	53	18.2
	Primary	30	10.3
	Secondary	155	53.3
	Tertiary	53	18.2
Hypertensive Status	Negative	219	75.3
	Positive	72	24.7
Gender	Male	136	46.7
	Female	155	53.3
Marital Status	Single	169	58.1
	Married	122	41.9
Drugs	Yes	235	80.8
	No	56	19.2
Diet Consumption	Yes	222	76.3
	No	69	23.7
Liver status	Affected	62	21.3
	Not affected	229	78.7
		Mean	**Standard Deviation**
Age		36.68	13.98
Liver size		6.95	1.76

Source: Authors' compilation.

Table 4.1 presents the descriptive statistics of the covariates considered in this study. The results revealed that majority, 141(48.5%) of the patients had hepatitis B, 81(27.8%) had hepatitis C, and 69(23.7%) of the respondents had hepatitis A. Additionally, majority 155(53.3%) of the patients had secondary school level of education, 53(18.2%) had tertiary, 30(10.3%) had primary and 53(18.3%) had no formal education. In addition, 219(75.3%) patients did not have hypertension, whereas 72(24.7%) had hypertension.

Furthermore, 136(46.7%) of the patients are male while 155(53.3%) were female. Based on marital status, 169 (58.1%) were single and 122 (41.9%) were married. In addition, 235(80.8%) of the patients were prescribed drugs, whereas 56 (19.2%) were not prescribed drugs. Based on diet consumption, 222(76.3%) patients who consumed good diet died, whereas 69(23.7%) did not. In addition, 62(21.3%) of the patients had liver damage, whereas 229(78.7%) did not.

The average age of the patients was 36.68 years with a standard deviation of 13.98 years. The average viral load was 398361.08 years with a standard deviation of 445374.3. Similarly, the average liver size of the patients was 6.95 with a standard deviation of 1.76.

5. Data Analysis and Results

Table 4.2: Proportional Hazard Assumption

Variable	Chi-Square	P-value
Hepatitis Type	0.36	0.5470
Educational Level	0.79	0.3750
Hypertensive Status	0.85	0.3557
Gender	0.48	0.4874
Marital Status	2.54	0.1107
Drugs	0.63	0.1774
Diet Consumption	2.73	0.4271
Liver status	0.34	0.2872
Age	1.82	0.0984
Liver size	0.07	0.7965
Global Test	10.89	0.4525

Source: Authors' compilation.

Table 4.2 presents the results for the proportional hazard assumption. From the results, it was observed that all covariates, including the global test, satisfied the constant hazard assumption, as evidenced by the p-value > 0.05. Thus, the proportional (constant) hazard assumption was satisfied.

Table 4.3: Model Comparison

Regression Moel	Baseline Distribution	AIC	BIC
PH	Exponential	147.1171	202.217
	Weibull	**113.9035**	**171.6766**
	Gompertz	121.4703	180.2435
AFT	Exponential	147.1171	202.217
	Weibull	113.9035	172.6766
	Lognormal	**112.4452**	**171.2184**
	Log logistics	112.8056	171.5788

Source: Authors' compilation.

Table 4.3 presents the model comparison using the AIC and BIC selection criteria. From the results, it was observed that among the three PH models considered in this study, the Weibull PH model outperformed the exponential and Gompertz PH models due to the least values of AIC and BIC. However, among the four AFT models considered in this study, the lognormal AFT model performed better than the other three (exponential, Weibull and log logistics AFT models) as occasioned by the AIC and BIC. Overall, the lognormal AFT model

had the lowest values of AIC and BIC, indicating that it outperformed all the other six models considered in this study.

Table 4.4: Parameter Estimate for Weibull PH Model

Covariates	Category	B	HR	SE	Z	P-value
Hepatitis Type	Hepatitis Type A	***Ref***				
	Hepatitis B	-0.5425	0.5813	0.3924	-0.80	0.422
	Hepatitis C	0.2593	1.2960	1.1958	0.28	0.779
Educational Level	No Education	***Ref***				
	Primary	-1.2787	0.2784	0.1997	-1.78	0.075
	Secondary	-0.5938	0.5522	0.2936	-1.12	0.264
	Tertiary	-0.1096	0.8962	0.7836	-0.13	0.900
Hypertensive Status	Negative	***Ref***				
	Positive	-0.2249	0.7986	0.3109	-0.58	0.563
Gender	Male	***Ref***				
	Female	-0.8368	0.4331	0.2657	-1.36	0.173
Marital Status	Single	***Ref***				
	Married	-0.6537	0.5201	0.3425	-0.99	0.321
Drugs	Yes	***Ref***				
	No	1.2992	3.6662	1.9410	2.45	0.014
Diet Consumption	Yes	***Ref***				
	No	0.7751	2.1709	1.0141	1.66	0.097
Liver status	Affected	***Ref***				
	Not affected	-2.6121	0.0734	0.0620	-3.09	0.002
Age		0.0223	1.0225	0.0216	1.05	0.293
Liver size		-0.2788	0.7567	0.1379	-1.53	0.126

Source: Authors' compilation.

The estimated covariates for the Weibull PH model are presented in Table 4.4. The Hazard Ratio (HR), which is simply the relative risk, was averaged during the trial. The HR < 1 indicates reduction in hazard, HR > 1 indicates an increase in hazard, and HR = 1 indicates no hazard.

The B column provides the estimate of the regression parameters with the interpretation that the estimated coefficient of B_{irs} gives the expected changes in the log hazard ratio for every one unit increase in the corresponding covariates X_{irs} when all other covariates are held constant. In the selected model (Weibull proportional hazard), the variables drugs and liver status were significant factors, implying that drugs and liver status were the two factors that significantly contributed to the hazard of liver cirrhosis. However, the variable hepatitis, educational level, hypertension status, gender, marital status, diet consumption, age and liver size were not significant factors.

The estimated coefficient for patients not on drugs was significant (B = 1.2992, HR = 3.6662, P < 0.05). HR > 1 implies that patients who were not on drugs have an increased risk of dying from liver cirrhosis compared with those who were on drugs (reference category). In addition, the estimated coefficient for patients whose livers were not affected was significant at the 5% level (B = -2.6121, HR = 0.0734, p < 0.05). The HR < 1 implies that patients whose livers were not affected have a lower risk of dying from liver cirrhosis than those whose livers were affected.

The estimated coefficients for hepatitis types B and C were not significant (B = -0.5425, HR = 0.5813, p> 0.05; B = 0.2593, HR = 1.2960, p > 0.05), respectively. The HR <1 for hepatitis type B implies that hepatitis B patients have a decreased hazard of dying from liver cirrhosis compared with hepatitis A patients, whereas hepatitis C patients have an increased hazard of dying from liver cirrhosis compared with hepatitis type A patients.

The estimated coefficient for patients with primary education level (B = -1.2787, HR = 0.2787, p > 0.05) and secondary school level (B = -0.5938, HR = 0.5522, p > 0.05) were not statistically significant. The HR for all three categories was 1, implying that patients with primary, secondary, and tertiary liver cirrhosis had a lower hazard of liver cirrhosis than those with no formal education (reference group).

The estimated coefficient for patients with a positive hypertension status was statistically insignificant (B = -0.2249, HR = 0.7986, p > 0.05). The HR < 1 implies that patients with a positive hypertension status have a lower risk of dying from liver cirrhosis than those with a negative hypertension status (reference group), although this difference is statistically insignificant. The estimated coefficient for female patients was statistically insignificant (B = -0.8368, HR = 0.4331, p > 0.05). The HR < 1 implies that female patients have a decreased hazard of dying from liver cirrhosis as compared to male (reference group), although this difference is statistically insignificant.
The estimated coefficient for patients who were married was statistically insignificant (B = -0.6537, HR = 0.5201, p > 0.05). The HR < 1 implies that patients who were married have a lower risk of dying from liver cirrhosis than those who were single (reference group), although statistically insignificant. The estimated coefficient for patients who did not consume a diet was statistically insignificant (B = 0.7751, HR = 2.1709, p > 0.05). The HR > 1 implies that patients not receiving diet have an increased hazard of dying from liver cirrhosis compared with those who were receiving diet (reference group), although this difference is statistically insignificant.
The estimated coefficient for patient age was statistically insignificant (B = 0.0223, HR = 1.0225, p > 0.05). HR > 1 implies that an increase in the age of patients increases the hazard of dying from liver cirrhosis, although this finding is statistically insignificant. However, the estimated coefficient for liver size was statistically insignificant (B = -0.2788, HR = 0.1379, p > 0.05). The HR < 1 implies that an increase in liver size decreases the hazard of dying from liver cirrhosis, although statistically insignificant.

Table 4.5: Parameter estimation for the lognormal AFT model

Covariates	Category	B	TR	SE	P-value
Hepatitis Type	Hepatitis Type A	***Ref***			
	Hepatitis B	0.3698	1.4474	0.2495	0.138
	Hepatitis C	0.0944	1.0990	0.2910	0.746
Educational Level	No Education	***Ref***			
	Primary	0.5569	1.7453	0.2938	0.058
	Secondary	0.1528	1.1651	0.1814	0.400
	Tertiary	-0.0415	0.9593	0.2814	0.883
Hypertensive Status	Negative	***Ref***			
	Positive	-0.0239	0.9764	0.1615	0.883
Gender	Male	***Ref***			
	Female	0.2588	1.2954	0.1795	0.149
Marital Status	Single	***Ref***			
	Married	0.0860	1.0898	0.2271	0.705
Drugs	Yes	***Ref***			
	No	-0.3732	0.6885	0.1707	0.029
Diet Consumption	Yes	***Ref***			
	No	-0.4476	0.6392	0.1653	0.007
Liver status	Affected	***Ref***			
	Not affected	0.8304	2.2942	0.2382	0.000
Age		-0.0016	0.9984	0.0086	0.850
Liver size		0.0674	1.0697	0.0553	0.223

Source: Authors' compilation.

Table 4.5 summarizes the results of the analysis of the log-normal AFT model. Covariates with TR < 1 indicate decrease in survival time of the patients while those with TR > 1 indicate an increase in survival time. The covariates drugs and liver status were significant predictors of survival in patients with liver cirrhosis.
The estimated coefficient for drugs was significant (B = -0.3732, TR = 0.6885, p < 0.05). The TR<1 indicates that patients that patients that were not on drugs had a shorter survival time compared with those who were on drugs (reference category). In addition, patients whose livers were not affected had an increased survival time compared

with those that were affected (B = 0.8304, TR = 2.2942, p < 0.05). This implies that patients with liver disease have a shorter survival time.

The covariates hepatitis type, educational level, hypertension status, gender, marital status, diet consumption, age, and liver size were not statistically significant in predicting the survival time of patients with liver cirrhosis at 5%.

The estimated coefficients for hepatitis types A and B were not significant (B = 0.3698, TR = 1.4474, p > 0.05; B = 0.0944, TR = 1.0990, p > 0.05), respectively. TR > 1 indicates that hepatitis types A and B increase the survival time of patients with liver cirrhosis compared with type A (reference group).

The estimated coefficients for educational level of liver cirrhosis patients were not significant at 5% (B = 0.5569, TR = 1.7453, p > 0.05; B = 0.1528, TR = 1.1651, p > 0.05; B = -0.0415, TR = 0.9593, p > 0.05), respectively. TR > 1 for primary and secondary school levels indicates that liver cirrhosis patients with those levels of education have increased survival times as compared with those with no formal education (reference category), whereas tertiary level have TR < 1 indicating that those with tertiary level education have decreased survival times as compared with those with no formal education (reference category).

The estimated coefficients for patients with liver cirrhosis whose hypertension status was positive were not statistically significant in predicting survival times (B = -0.0239, TR = 0.9764 p > 0.05). The TR < 1 though statistically insignificant, indicates that liver cirrhosis patients with a positive hypertension status had a shorter survival time than those with a negative hypertensive status.

Females have an increased survival time compared with males (B = 0.2588, TR = 1.2954, p> 0.05), although statistically insignificant. Similarly, patients with liver cirrhosis who were married had increased survival times compared with those who were not married (B = 0.0860, TR = 1.0898, p > 0.05). In addition, an increase in liver size increased the survival time of the patients (B = 0.0674, TR = 1.0697, p> 0.05).

However, patients without a good diet had a shorter survival time than those with a good diet, although this difference was statistically insignificant (B = -0.4476, TR = 0.6392, p > 0.05). Similarly, an increase in age tended to decrease the survival times of patients with liver cirrhosis (B = -0.0016, TR = 0.9984, p > 0.05), although statistically insignificant.

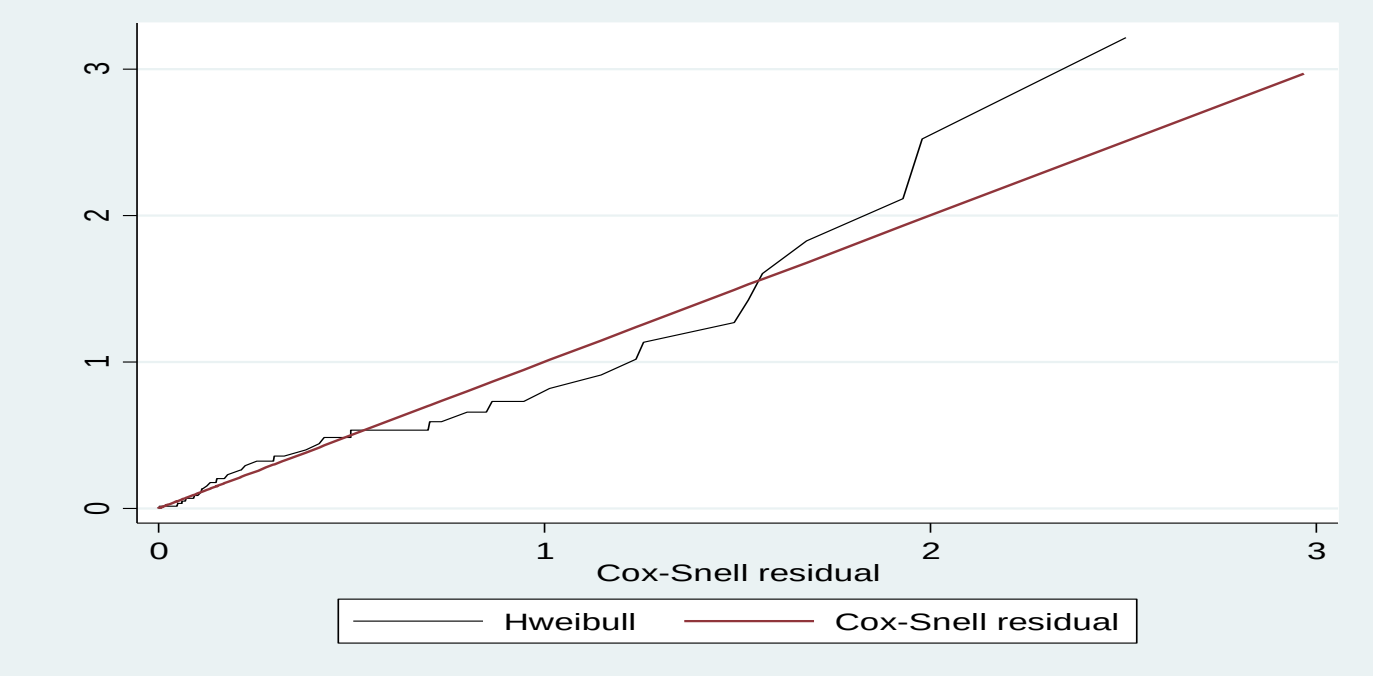

Figure 4.1: Overall model fit for the Weibull PH model.

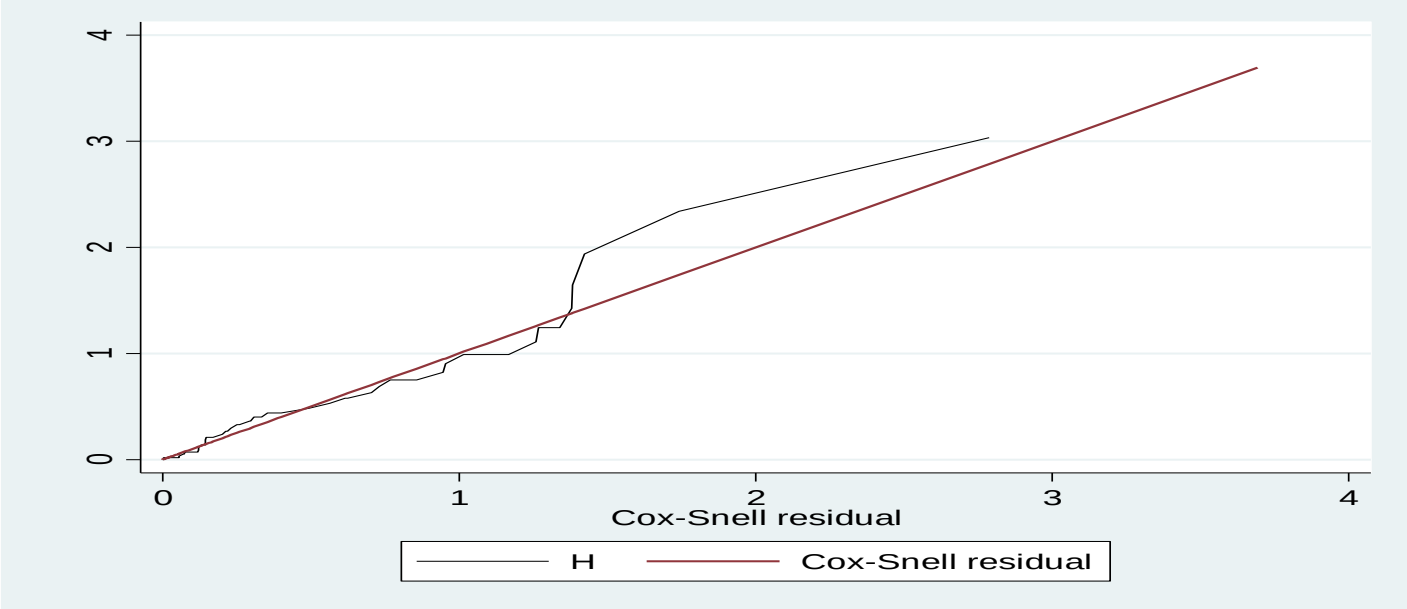

Figure 4.2: Overall model fit for lognormal AFT model.

Cox–Snell residuals are one way to investigate the overall fit of the fitted models to data. The plots for the fitted residuals for the selected parametric Weibull PH model and parametric log-normal AFT model are presented in Figures 3.1 and 3.2, respectively. If the model fits the data well, then the plot of cumulative hazard function should line up with the Cox–Snell residuals. The plots for both models did not show a strong deviation, as they tended to make a straight line through the origin with little deviation at the tail end, suggesting that the selected models are appropriate for time-to-event analysis of liver cirrhosis data.

6. Conclusions

In this study, the performance of the AFT and PH models with some baseline distributions was examined using the frequentist approach. The right-hand censored liver cirrhosis data were used. Specifically, the performance of the parametric AFT model with four baseline distributions (exponential, Weibull, lognormal and log logistics) and the parametric PH model with three baseline distributions (exponential, Weibull and Gompertz) was evaluated. The results of the model comparisons indicated that the lognormal AFT model had the lowest AIC and BIC values and was selected as the optimal model. This finding agrees with the findings of a previous study by Shankar et al. (2019).

The results from the optimal model (lognormal AFT model) revealed that liver cirrhosis that were not on drugs have decrease survival times. This implies that patients on drugs have longer survival times. In addition, liver status was a significant predictor of survival in patients with liver cirrhosis. Patients with liver disease have a shorter survival time. This indicates that patients with liver cirrhosis whose liver is not affected have an increased survival time.

The overall model fit was evaluated using Cox-Snell residuals. The plots indicated a good fit as there was no strong deviation of the cumulative hazard from the Cox- Snell residuals.

7. Summary

The aim of this study was to fit an appropriate parametric survival model to right-censored liver cirrhosis data using the frequentist approach. The study used secondary data. The data were examined for proportional hazard assumptions, and the results revealed that the constant hazard assumption was met as the p-value for the global test was greater than 0.05. In order to achieve the objective, three PH models were compared (Exponential,

Weibull and Gompertz). The results revealed that the Weibull PH model outperformed the exponential and Gompertz PH models because it had the lowest AIC and BIC.

Similarly, to achieve the two objective, four different AFT models (expandential, Weibull, logistic, and lognormal) were compared. The results demonstrate that the Lognormal AFT model outperformed the other three AFT models. The overall performance of the models (PH and AFT) was also evaluated. The results revealed that the lognormal AFT model had the least AIC and BIC values and hence outperformed the PH model.

Based on the overall performance model, it was discovered that the drugs and liver status were significant predictors of survival in patients with liver cirrhosis.

8. Conclusion

The current study fitted an appropriate parametric model to liver cirrhosis data. The proportional hazard model and the accelerated failure time model were also considered. Based on the findings of this study, it was concluded that the lognormal accelerated failure time model performed better than the other models considered in this study. Based on this model, we concluded that drugs and liver status were significant predictors of survival in patients with liver cirrhosis.

9. Recommendations

Based on these findings, the following recommendations were made:

i. The liver cirrhosis patients who were on drugs should adhere strictly to their medication as it increases their survival time.

ii. According to these findings from this study, patients with liver cirrhosis should always carry out a liver function test to know the status of their liver and ensure that it is in good state.

10. Limitations

This study relied on secondary data obtained from an existing source; thus, the data may not be reliable. In addition, the study focused on a parametric survival model with a special interest in classical distribution; this model might not be sufficiently flexible to capture the hazard shapes in the data.

11. Suggestions for Further Studies

Based on the limitations of the study listed above, it was suggested that further studies employ flexible distributions capable of capturing different hazard shapes that exist in survival data.

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Appendix A: Liver Cirrhosis Data

Time	Status	HT	EDUL	HYPSTATUS	Sex	Marital Status	Age	Drugs	Diet	Liver status	Liver size
90	0	2	3	1	2	2	35	2	1	1	5.8
121	0	2	3	1	1	1	33	1	2	1	8
30	1	3	3	2	2	1	29	2	2	1	6.2
30	0	1	4	1	2	2	30	1	1	2	6.45
60	0	1	4	1	1	1	31	1	1	2	9.55
30	0	3	3	2	1	1	21	2	2	1	7.32
60	1	2	3	1	1	1	24	1	2	1	4.48
31	0	3	4	1	2	2	33	1	1	1	5.14
30	0	3	3	1	1	1	29	1	1	1	8.42
60	1	1	3	1	1	1	30	2	2	1	10.1
90	0	2	3	1	1	1	24	1	1	1	7.63
60	0	2	3	1	2	2	28	1	1	2	6
30	0	3	2	2	2	1	22	2	2	1	4.13
30	0	2	3	1	2	1	18	1	1	2	6.31
60	0	1	4	1	1	2	33	1	1	2	9.5
90	0	1	3	1	2	1	31	1	1	2	6.42
30	0	2	3	2	1	1	36	1	1	2	8.25
30	0	2	3	1	2	2	32	2	2	1	5.55
90	1	2	3	2	1	1	29	2	2	1	5.49
90	1	2	2	1	2	1	24	2	2	1	3.14
150	0	2	3	1	1	2	33	1	1	1	8.66
181	1	3	3	2	1	1	24	1	2	1	6.3
2	0	1	3	1	2	1	29	1	1	1	5.5
91	0	2	2	1	2	1	21	1	1	1	6.15
24	0	2	3	1	1	2	30	1	1	2	7.23
29	0	2	3	1	1	1	31	2	2	1	9
61	0	1	4	1	2	1	29	2	2	1	6.55
29	0	2	3	1	1	2	30	1	1	2	8.4
31	0	2	3	1	1	1	23	1	1	2	9
92	0	2	4	1	2	1	30	1	1	2	6.27
62	0	2	3	1	1	1	29	1	2	2	8.4
31	1	3	3	2	2	2	40	2	2	1	3.14

30	0	3	4	1	1	2	60	1	1	2	5.41
61	0	1	3	1	1	2	50	1	1	2	9.6
31	0	3	3	1	1	1	23	1	1	1	4.15
91	0	2	4	1	2	2	45	1	1	2	7
30	0	2	3	1	2	1	29	1	1	2	7
31	0	2	3	1	1	1	30	1	1	2	7.65
62	0	2	3	2	2	2	70	1	1	2	8.6
61	0	2	4	1	1	2	60	1	1	2	6.75
30	1	1	3	1	2	1	31	1	2	1	9.6
61	0	1	3	1	2	1	35	1	1	2	2.36
58	0	2	3	1	1	1	30	1	1	2	5.5
28	0	2	3	1	2	2	32	1	1	2	10
61	0	3	3	1	1	2	47	1	1	2	6.92
29	0	2	3	1	1	1	24	2	1	2	9.36
59	0	2	3	1	1	1	24	2	1	2	9.3
119	0	2	2	1	1	2	46	1	1	2	6.45
60	0	2	1	1	2	1	21	2	1	1	5.15
31	1	3	1	1	2	1	23	2	2	1	4.75
92	0	3	3	1	2	1	22	1	2	1	3.45
61	0	2	4	2	1	2	63	1	1	2	8.4
60	0	1	3	2	1	2	60	1	1	2	10.15
94	1	1	1	2	2	2	33	2	2	1	2.64
62	1	2	1	2	2	1	40	2	2	1	3.14
64	0	3	2	1	2	2	61	1	1	2	6.43
59	0	3	1	1	1	1	28	2	1	2	8.64
22	1	3	1	1	1	1	25	2	2	1	5.3
61	0	2	3	1	1	2	61	1	1	2	9.66
15	0	2	1	1	2	2	51	1	1	2	7
32	0	3	1	1	1	1	29	1	1	2	8.55
65	0	1	4	1	2	2	57	1	1	2	6.89
67	0	1	3	1	1	1	46	1	1	2	8.96
92	1	2	3	2	2	1	36	2	2	1	3.75
30	0	2	1	1	1	2	70	1	1	2	10
121	0	2	1	2	2	2	66	1	2	1	5.2
62	0	1	3	1	1	1	19	1	2	2	8.3
29	0	1	2	1	1	1	12	1	1	1	7.05
32	0	3	3	1	2	1	21	1	2	1	4.2
31	1	1	3	2	1	2	71	1	2	1	4.16
72	1	1	1	1	1	2	60	2	2	1	8.14
63	0	2	2	1	2	1	16	2	1	1	3.3
28	0	3	3	2	1	2	74	1	1	2	8.2
91	0	3	3	2	2	2	64	1	1	2	6.43
91	0	2	4	1	2	1	30	1	2	2	6.01
64	0	1	3	1	1	1	33	1	1	2	9.1
32	0	2	4	1	1	1	24	1	1	2	9.47

63	0	2	4	1	2	2	36	1	1	2	6.49
34	1	2	3	2	1	2	50	2	2	1	5.14
93	1	1	3	1	2	1	41	1	1	1	3.6
62	0	3	3	1	2	1	35	2	1	1	5.48
32	0	2	1	1	1	2	51	1	1	2	8.4
122	0	2	1	1	1	2	62	1	2	2	9.5
93	0	3	3	1	2	1	41	1	1	2	6.02
61	0	2	3	1	2	1	24	1	1	2	6.43
33	1	3	1	2	1	1	30	1	2	1	7.03
32	0	3	1	1	2	2	70	1	1	2	5.2
61	0	2	3	1	1	1	25	1	1	2	9.3
64	0	2	3	1	2	2	49	1	1	2	5.41
29	0	2	3	1	1	1	23	2	2	2	8.32
93	0	1	1	1	1	1	21	1	1	2	9.01
61	0	1	3	2	2	2	50	1	1	2	6
93	0	1	2	1	2	1	20	1	1	2	6.57
62	0	3	3	1	2	1	22	1	2	2	5.47
61	0	3	1	1	1	2	50	1	1	2	7.49
29	0	3	1	1	1	1	30	1	2	2	8.04
29	0	2	3	2	2	2	60	1	1	2	5.5
98	0	2	4	1	1	2	70	1	1	2	9.05
62	0	2	4	1	1	2	55	1	1	2	8.31
60	0	2	3	1	2	1	31	2	1	2	6.48
91	0	2	1	1	2	2	46	1	2	2	6.02
92	0	1	2	2	2	1	28	1	2	2	5.92
92	0	1	1	2	1	1	24	1	2	2	8.06
30	0	1	3	1	1	2	49	2	1	2	8.46
32	0	2	1	1	1	2	50	1	1	2	8.92
28	0	2	3	1	1	1	31	1	1	2	8.46
29	0	3	3	1	1	1	33	1	1	2	7.98
93	0	3	3	2	2	2	48	1	1	1	6.15
61	0	3	3	1	2	2	51	1	1	2	6.05
91	0	2	4	1	2	1	30	1	2	2	6.32
91	1	2	1	2	1	2	41	1	2	2	4.3
62	1	2	3	2	1	1	29	1	1	1	5.15
61	0	1	4	1	2	1	27	2	1	1	6.26
57	0	1	3	1	2	2	50	1	1	2	6
90	0	1	3	1	1	1	24	1	1	2	7.42
91	0	3	3	1	1	1	21	1	2	2	8.66
61	0	2	3	1	2	2	50	1	1	2	5.2
90	0	2	3	1	1	2	54	1	1	2	8.45
93	1	1	2	2	2	1	28	2	2	1	3.34
45	0	3	1	2	2	1	24	1	1	2	5.9
30	0	2	1	1	1	2	36	2	1	2	9.25
30	0	2	3	1	1	2	41	1	1	2	8.52

91	0	3	3	1	2	2	47	1	1	2	6.04
89	1	3	2	1	2	1	27	2	2	1	3.15
91	0	2	2	1	2	1	25	2	1	2	5.26
60	0	1	1	1	2	1	22	1	1	2	6.3
59	0	1	1	1	1	2	50	1	1	2	9.27
30	0	1	3	2	1	1	29	1	1	2	8.15
60	0	3	4	1	2	2	54	1	1	2	5.58
30	0	2	3	1	1	2	61	1	1	2	9.52
29	1	3	3	2	1	1	34	2	2	1	4.55
83	0	3	4	2	1	2	70	1	1	2	9.47
89	0	2	3	1	1	1	31	2	1	2	8.7
60	0	3	3	1	2	1	37	2	1	2	5.3
62	0	1	4	1	1	2	66	1	1	2	8.92
92	0	1	2	1	1	1	31	1	2	2	7.42
93	0	3	1	1	2	1	31	1	1	2	5.83
61	0	2	3	2	2	2	51	1	2	2	6.87
61	0	2	1	1	2	1	30	1	2	2	5.4
92	0	2	3	1	1	2	47	2	1	2	8.03
96	0	3	3	1	1	2	50	1	1	2	8.53
92	0	1	1	1	1	1	24	2	1	2	7.44
61	0	2	1	1	1	1	21	1	1	2	8.49
61	0	2	3	2	2	2	54	1	1	2	6.52
29	0	2	3	1	2	1	31	1	1	2	6.66
30	0	2	3	1	1	2	41	1	1	2	9.6
60	0	2	3	1	1	2	36	1	1	2	8.4
92	1	2	1	1	2	1	17	2	2	1	3.14
90	0	3	3	2	2	1	28	1	1	2	6.47
29	0	3	1	1	2	2	47	1	1	2	6.14
27	0	2	4	1	2	2	50	1	1	2	5.58
90	0	2	3	2	2	1	31	1	1	2	6.01
62	0	1	3	1	1	1	30	1	1	2	8.67
61	1	3	1	2	1	1	27	2	1	1	5.27
183	0	3	3	1	1	2	60	1	1	2	9.87
123	0	2	4	1	2	2	70	1	1	2	6.2
91	0	1	3	2	1	1	25	1	1	2	8.4
63	0	1	4	1	2	2	67	1	1	2	6.38
61	1	1	2	1	2	1	29	2	2	1	4.32
92	0	2	3	1	1	1	36	1	1	2	9.4
32	0	1	3	2	1	2	55	1	1	2	8.66
29	0	2	2	1	2	1	30	1	1	2	6.09
28	1	3	1	1	2	1	26	2	2	1	3.59
90	0	3	3	1	1	1	24	1	1	2	8.33
91	0	3	3	1	2	2	49	1	1	2	6.11
92	0	2	2	2	2	1	29	2	1	2	5.92
62	0	2	3	1	1	1	33	1	1	2	8.92

121	0	1	4	1	2	2	52	1	1	2	9.7
120	0	1	3	1	2	1	41	1	1	2	6
52	0	3	3	1	1	1	38	1	1	2	9.25
31	1	3	1	2	2	1	23	2	1	1	4.01
29	0	2	4	1	2	2	37	1	1	2	6.4
93	0	2	3	2	2	1	24	1	1	2	6.75
93	0	2	3	2	1	1	19	1	1	2	9.2
91	0	2	3	1	2	2	40	1	1	2	6
93	1	2	4	1	2	2	43	1	1	2	6.4
60	0	2	2	1	1	1	31	1	2	2	9.2
94	0	1	3	1	1	1	27	1	1	2	8.44
87	1	2	1	2	2	1	24	2	1	1	3.14
28	0	1	3	1	2	2	49	1	1	2	7
90	0	2	3	2	2	2	50	1	1	2	6.52
87	0	2	4	1	1	1	33	1	1	2	8.95
64	0	1	3	1	1	1	36	1	1	2	8.53
60	0	2	1	1	2	1	25	1	1	2	6.09
92	1	3	3	1	2	1	31	1	2	1	6.47
90	0	2	3	1	1	2	55	1	1	2	9.8
91	0	2	1	1	2	1	34	1	1	2	6.4
66	0	2	3	1	2	2	52	1	2	2	5.57
61	0	1	3	2	1	1	31	1	1	2	8.7
61	0	1	1	1	2	2	26	2	2	2	5.02
60	0	3	2	2	2	1	33	1	1	2	6.02
61	0	1	3	1	2	2	22	1	1	2	6.7
92	0	2	3	1	1	1	30	1	1	2	8.99
92	0	2	1	1	1	1	28	1	2	2	9.02
93	0	2	4	1	2	1	40	1	1	2	7
30	0	3	3	1	1	2	31	1	1	2	8.31
27	0	2	3	2	2	1	21	2	1	2	6.12
30	0	3	3	1	2	1	20	1	2	2	5.5
92	1	2	3	1	1	1	26	2	1	1	6.04
64	0	2	4	1	2	2	31	1	1	2	6.87
64	0	2	4	1	1	2	49	1	2	2	9.2
59	0	2	2	1	2	1	36	1	1	2	5.89
95	0	1	3	2	2	2	52	1	1	2	6.08
91	0	2	4	2	1	2	41	1	1	2	9.2
89	0	1	3	1	2	1	24	1	1	2	6.95
89	0	2	3	1	1	1	20	2	1	2	8.46
60	0	2	4	2	1	2	54	1	2	2	8.32
60	0	3	3	1	2	2	50	1	1	2	6.02
93	0	3	3	1	1	1	41	1	1	2	9.06
93	0	2	3	2	2	1	36	1	2	2	6.27
60	0	1	3	1	2	2	46	1	1	2	5.92
94	0	1	3	1	2	1	32	1	1	2	5.65

60	0	1	4	1	1	1	50	1	2	2	9.86
93	0	3	3	2	2	2	31	1	1	2	6.27
91	0	3	3	1	1	1	10	2	1	2	9.6
60	0	2	3	1	2	1	33	1	1	2	5.96
60	0	2	3	1	2	1	24	1	1	2	6.3
91	0	3	2	1	2	1	14	2	1	2	6.57
90	0	2	4	2	2	1	20	1	1	2	6.93
60	0	2	3	1	1	2	47	1	1	2	10
28	0	1	1	1	2	2	36	1	1	2	6.91
28	0	3	3	1	1	1	21	1	2	2	9.36
30	0	3	1	1	1	1	24	1	1	2	8.75
91	0	2	2	2	2	2	60	1	1	2	6.84
30	0	3	1	1	2	1	24	2	1	2	5.92
120	0	3	3	1	2	1	30	1	2	2	5.89
64	0	2	4	2	2	2	70	1	1	2	6.2
62	0	1	3	1	1	2	64	1	1	2	9.03
62	0	1	4	1	2	2	69	1	1	2	6.06
62	0	3	4	2	2	1	28	2	2	2	7
61	0	3	3	2	2	1	31	1	1	2	6.01
29	0	3	3	2	2	2	29	1	1	2	5.83
91	0	2	2	1	1	2	50	1	1	2	8.4
91	0	2	4	1	1	2	47	1	1	2	9.2
92	0	3	3	1	2	1	31	1	1	2	6.04
61	0	3	4	1	2	1	30	1	1	2	6.92
61	0	2	3	1	1	1	46	1	1	2	8
64	0	2	1	1	2	2	31	1	1	2	6
92	0	1	2	1	2	1	31	1	1	2	3.03
61	1	1	4	1	2	2	16	1	2	1	5.79
31	0	3	4	1	1	2	52	2	1	2	5.79
32	0	3	3	1	1	1	20	1	1	2	8.4
64	0	3	2	1	2	1	21	1	1	2	6.27
93	0	2	1	1	1	1	24	1	1	2	7.83
62	0	2	3	1	2	1	22	1	1	2	5.59
61	0	2	4	1	2	2	64	1	1	2	6.3
61	1	3	3	1	1	1	43	2	2	1	5.1
91	0	1	4	1	2	2	72	1	1	2	6.89
91	0	1	3	1	2	2	63	1	1	2	6.03
62	0	3	3	2	1	1	24	1	1	2	9.3
82	0	3	3	2	1	1	29	1	1	2	8.4
31	0	1	4	1	2	1	28	1	1	2	6.31
94	0	2	1	1	2	2	52	1	1	2	6.1
61	1	1	3	1	1	1	31	2	2	1	4.72
62	0	2	2	2	1	2	54	1	1	2	9.62
64	0	1	3	1	2	1	27	1	1	2	6.88
91	0	3	1	2	2	1	25	1	1	2	6.66

60	0	2	3	2	1	1	47	2	2	2	9.92
60	0	2	1	1	2	1	21	1	1	2	6.75
92	0	2	3	2	2	2	55	1	1	2	5.82
28	0	2	3	1	1	1	28	1	1	2	9.99
30	0	3	4	1	1	2	60	1	1	2	10.03
93	0	1	4	1	2	1	29	1	1	2	6.4
58	0	2	3	1	2	2	42	1	1	2	6
62	0	2	3	1	1	1	28	1	1	2	8.43
93	0	3	3	2	1	2	48	1	1	1	7.2
61	0	3	2	1	1	1	32	1	1	2	8.39
91	0	2	4	1	1	2	50	1	1	2	7.59
93	0	2	2	1	2	1	22	1	1	2	6
91	0	2	1	1	2	1	27	1	1	2	5.43
31	0	3	3	2	2	2	37	1	1	1	5.1
32	0	1	3	1	1	1	25	1	1	2	8.15
92	0	1	1	1	1	1	29	1	1	2	7.55
93	0	1	3	1	2	2	33	1	1	2	5.15
62	0	2	3	1	2	1	30	1	1	2	6.15
92	0	3	1	2	2	2	36	1	1	1	5.02
62	0	3	2	1	2	1	24	1	1	2	6.01
61	0	2	4	1	1	2	41	1	1	2	10.01
92	0	2	2	1	1	1	30	1	1	2	7.59
57	0	2	1	2	2	1	28	1	1	1	4.13
93	0	2	4	1	2	2	36	1	2	2	6.15
61	0	1	3	2	1	1	22	1	1	2	8.43
63	0	3	3	1	1	1	24	1	2	2	8.22
92	0	3	4	2	2	2	39	1	2	1	4.32
62	0	2	3	1	1	1	30	1	2	2	8.47
62	0	2	4	1	1	2	36	1	1	2	9.34
63	0	2	3	1	2	2	42	1	1	2	6.49
152	0	2	3	2	2	1	28	1	1	1	4.59
92	0	2	1	1	2	1	25	1	1	2	6.3
63	0	2	2	1	2	2	40	1	1	2	6.42
62	0	2	4	2	2	1	21	1	1	2	7

Source: Selected Hospitals