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CONDITIONAL VOLATILITY OF CRUDE OIL PRICES IN NIGERIA USING GENERALIZED AUTOREGRESSIVE CONDITIONAL HETEROSKEDASTICITY MODELS

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Abstract
This study investigates the conditional volatility of crude oil prices in Nigeria using Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models and their variants. Given the significant economic dependence of Nigeria on crude oil revenues, understanding the dynamics of fluctuations in oil prices is critical. This study analyzes monthly and daily crude oil price data from 2006 to 2022, employing ARIMA-GARCH, TARCH, and other heteroskedasticity models under different error distributions. Statistical tests, including stationarity checks, normality measures, and heteroskedasticity diagnostics, were conducted to ensure model adequacy. Results reveal strong evidence of volatility clustering and long memory effects in the return series, with asymmetric models like TARCH outperforming symmetric GARCH in capturing leverage effects. Furthermore, volatility mean reversion and half-life analysis indicate that shocks in oil price returns persist over time but gradually revert to mean values. These findings underscore the necessity of accurate volatility modeling to enhance forecasting, risk management, and policy formulation in oil-dependent economies like Nigeria.

INTRODUCTION

Crude oil is a naturally occurring unrefined petroleum product composed of hydrocarbon deposits and other organic materials. Crude oil can be refined to produce usable products such as gasoline, diesel and various forms of petrochemicals. It is a nonrenewable resource, also known as a fossil fuel, which means that it cannot be replaced naturally at the rate it is consumed; therefore, it is a limited resource.

The trend and dwindling of oil prices in the global market have its price in the global market has become a source of concern for oil-producing countries. The price of crude oil dropped precariously from a peak of $104

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per barrel by the third quarter of 2014. Specifically, the OPEC average monthly basket price of oil peaked at \$107.89 per barrel in June. 2014 dwindled very sharply to \$59 per barrel at end- December, 20M. It further decelerated to \$5<4 by end March, 2015, and the instability in the price continues till date, resulting in Nigeria experiencing a sudden and significant drop in revenue inflow from oil sales. Nigeria, a mono-cultural and hydrocarbon economy, depends largely on revenue realized from oil to sustain its teeming population and the economy in order to foster physical, political, and socioeconomic development.

The price of crude oil suffers sudden fluctuations, and this continual fluctuation affects many things, thereby contributing to the increase in price volatility and revenue profile of these products. These arcs are some causes of economic shocks that are widely experienced in the world.

Over the years, the study of oil prices and volatility has remained one of the most important economic trends in terms of increasing investment and minimizing risk. Therefore, it necessary to use an accurate statistics method to know the changes in price in terms of increase and decrease through what is known as long memory (Mostafaei and Sakhabakhsh, 2012). Long memory is a phenomenon we may sometimes face when analyzing time-series data where long-term dependence between two points increases the amount of distance between them (Bahar *et al..*. 2017). “Usually when modeling long memory behavior for any time series, such as mathematics, economics, among others; the operation can be more accurate by relying on the Generalized Autoregressive Conditional Hcteroskedacity (GARCH) models compared with Autoregressive integrated Moving Average (ARIMA) models.” It can also have an important impact in the financial field (Bhardwaj and Swanson, 2006), where long memory models are one of the most important models used in the analysis of time series (Karia *et al.*, 2013). The GARCH model is fitted to the time series data either to better understand the data or to predict future points in the series (forecasting). The use of forecasting in statistics and financial fields is essential at the national, regional, and international levels. It helps investors reduce financial risks and increase profits during volatility in the global economy. The GARCH model was introduced by Bollerslev (1986), as mentioned by Mostafaei and Sakhabakhsh (2012), to capture the long memory behavior of this time series data. The long memory feature exists if the autocorrelation function (ACT) decays more slowly than the exponential decay described by Bahar *et al.* (2017) or detected by using statistical methods, namely Hurst Exponent, as explained in Beran (1994). In addition, it is a known fact that long memory characteristics observed in data can be generated by a nonstationary structural break, as mentioned by (Ohanissian *et al.* 2008). Therefore, the importance of testing for structural breaks in the conditional mean of a time series is necessary, as it determines that long memory is real or fake, as pointed out by Vanessian *et al.* (2008). Therefore, the break detection procedure exhibits desirable properties both in the presence of breaks (stable potency across multiple breaks), as pointed out by Pretis *et al.* (2016). Besides, performing structural break testing when estimating the GARCH model is of great importance as it increases accuracy and prediction confidence. On the other hand, volatility is an important consideration for any time series, especially for oil prices. Volatility is noticeable in studies related to financial, economic, tourism, and other areas, where data are widely scattered (Tendai and Chikobvm 2017; Akter and Nobi, 2018). As is known, there are obvious volatilities shown in some types of time scries especially in crude oil prices (Lee and Huh, 2017). Therefore, it was necessary to study these volatilities to avoid-inaccuracies in the development, of plans and strategies for making important decisions or for future predictions necessary. Moreover, to mow their impact when forecasting to avoid any financial risks that may cause fosses to the investor as the forecasting of financial time series data is yet as one of the most difficult tasks due to the non-stationary and non-linearity, as studied by Ismail and Awajan (2017). Also, Ramzan *et al.* (2012) showed that one category of models that has confirmed successful in forecasting volatility in many cases is the GARCH Variants family of models. This was studied using the Generalized Autoregressive Conditionally Heteroskedastic (GARCH) model and Exponential! Generalized Autoregressive Conditionally Heteroskedastic (EGARCH) model. Based on the reasons above, we are choosing to study long memory and volatility in this study, due to the modality of Brent crude oil prices grow exponentially, non-stationery, and arc volatile. These phenomena are popular features found in much large-scale data.

Hence, this study, among other things, adds to the existing literature in several important ways. It is on this ground that the study will model volatility of crude oil prices from 2000 to 2020 using ARIMA-GARCH Variants models to determine the long-term trend and fluctuations in tire prices of crude oil. 1'his will help reveal the patterns exhibited by the prices of crude oil over time.

Statement of the Problem

Owing to the possible effects of crude oil prices on the economy, it is the interest of thirsty to carry out an analysis of the long-term trend and fluctuations in crude oil prices. This will help reveal the patterns exhibited by the prices of crude oil over time. As a result, the analysis of crude oil prices has been extensively researched by Statisticians and researchers in other fields. Economists in the energy complex have used several methods to forecast crude oil prices, including future prices, structural models, and time series techniques (Gray and Tomek, 2017). However, comparison on performance of volatility heteroskedasticity models has not been extensively considered for the price of crude oil.

This study, therefore, provides an empirical performance comparison of different Heteroskedasticity models on volatility return of crude oil prices in Nigeria, which has not been established in the literature.

Objectives of the Study

i. To test for Stationarity and Hetcroscedasticity:
ii. To fit appropriate Heteroskedasticity ARIMA-GARCH Variants:
iii. To estimate volatility, mean reversion, and half-life of crude oil returns in Nigeria

Scope of the Study

This study intends to concentrate on modeling the price of crude oil in Nigeria using ARIMA- GARCH Variants. The volatility effect on model performance will also be studied. Only the first and second orders of autoregressive and moving average of each model are considered. The data will be analyzed to examine whether they are stationary or not using the Augmented Dickey Fuller (ADI7) unit root test. In addition, the heteroskedasticity phenomenon will be tested for various data collected across months and years.

Information criteria, such as Alkaikc information criteria, Bayesian, and Hannah-Quinn information criteria, will be used to fit the data with the aim of selecting the best model. The adequacy of the selected models will then be determined for future forecasting using accuracy measures such as root mean, square error (RMSE), mean absolute percentage error, (MAPE), and TIC. The data will be analyzed using the R Software package.

Empirical Review

Following the seminal work of Engle (1982) on volatility modeling, several other studies have been conducted. Yet. Certain theoretical/empirical issue, such as the modeling Conditional Volatility of Crude oil prices in Nigeria using some hetcroskedasticity models, such as ARMA-GARCH variants of first and second order, is scarce and unresolved. Some of the works on volatility modeling estimate a particular GARCH model with one or two error distributions, while others apply a particular error distribution. Few ARCH family models are: (i) establish the best forecasting model for conditional variance: (ii) determine the best fitted volatility model; or; (hi) confirm the ability of a model to capture stylized facts inherent in high-frequency financial time series. As has been noted, the contribution of trade volume (as a surrogate for information arrivalto explain stock return volatility) and the error distribution assumption on the forecasting performance of returns volatility is very scanty (minimal). The available literature tends to capture the symmetric and asymmetric properties of financial data.

Salim (2013) employed GARCH family models and the Elman artificial neural network (ENN) system to model and predict crude oil volatility. Indeed, the study found that recurrent artificial neural networks were chosen in the studies to model and predict future crude oil price volatility data estimated by GARCH family models since they were nonlinear systems capable of learning noisy and nonstationary data. In particular, four hybrid systems were tested and compared; including the GARCH-ENN, EGARCH-ENN, APARCH-ENN, and TARCH-ENN systems. Using Brent crude oil price data, the obtained out-of-sample simulation results indicated that all hybrid systems provided very accurate forecasts of Brent future volatility. In addition, they showed evidence of the superiority of the GARCH-ENN system over the EGARCH-ENN. TARCH-ENN, and APARCH-ENN systems. The four proposed hybrid systems achieved very low forecasting errors. “Thus, they could be effective in oil industry management and applications.”

Achal *et al.* (2015) studied the autoregressive integrated moving-average (ARIMA) model, generalized autoregressive conditional heteroscedastic (GARCH) model, and exponential GARCH (EGARCH) model along with their estimation procedures for modeling and forecasting three price series—namely, domestic and international edible oil price indices and the international cotton price Tmlook A index. The Augmented Dickey-Fuller (ADF) and Philips Perron (PP) tests have. Been used for testing the stationary of the series. The Lagrange multiplier test was applied to detect the autoregressive conditional heteroscedastic (ARCH) effect. A comparative study of the above three models was conducted in terms of the root mean square error (RMSE) and relative mean absolute prediction error (RMAPE).

The residuals of the fitted models were used for diagnostic checking. This study has revealed that the EGARCH model outperformed the ARIMA and GARCH models in forecasting the international cotton price series, primarily due to its ability to capture the asymmetric volatility pattern in the data.

Thomas *et al.* (2015) used the Markov-switching multifractal (MSM) model and generalized autoregressive conditional Hetcroskedasticity (GARCH)-type models to forecast volatility in oil prices over the time periods January 02. 1875 to December 3D 1895 and January 03, 1977 to March 24, 2014. Based on six different loss functions and with the superior predictive ability (SPA) test, this study evaluated and compared forecasting performance for short and long horizons. The empirical results indicate that none volatility models could uniformly outperform other models across ah six different loss functions. However, the new MSM model was the model that most often across forecasting horizons and subsamples could not be outperformed by other models, with long memory GARCH-type models coming out second best.

Fredj (2016) investigated the dynamics of oil price volatility by examining interactions between the oil market and the US dollar/euro exchange rate. Unlike previous related studies that focused on low-frequency data and GARCH volatility measures, the present study used recent intraday data to measure realized volatility and investigate the instantaneous intraday linkages between different types and proxies of oil prices and US$/eurovolatilities. The specified drivers of oil price volatility through a focus on extreme USS exchange rate movements (intraday jumps). The study found a negative relationship between die US dollar/euro and oil returns, indicating that USS appreciation decreases oil prices. Secondly, the presence of a volatility spillover from the US exchange to the oil market. Interestingly, this spillover effect seems to occur through intraday jumps that occur simultaneously in both markets.

Omur *et al.* (2016) analyzed the return volatility of spot market prices of crude oil (WTI) and natural gas for two different terms which cover 02.01.2009 - 28.04.2014 and 04.01.2010- 28.04.2014 with different versions of the GARCH class models, such as GARCH, IGARCIL GJRGARCH, EGARCH, FIGARC'H, and FLAP ARCH. In particular, the main idea behind employing various GARCH models was to determine which one of these linear and nonlinear asymmetric models performs more accurately in terms of in-group and intergroup activities. Therefore, the main idea of the tire study was to determine a model that ensures to obtain a maximum return with response to the minimum loss for returns of the investments held by individual investors and fund managers, private sector budget planning decision makers, and state agencies forecasting macroeconomic indicators. To do so, the 10-day out-of-sample volatility forecasts of loss functions were considered to capture the forecasting performance of GARCH class models and to prevent forecasting errors with an efficiency hedge ratio in the energy market. For two periods, the asymmetric and Integrated GARCH models provide relatively a more accurate performance than other available models. For the first period, the minimum loss model was FIGARCH-BBM (SST) and for the second period, was EGARCH (GED) for WTI crude toiletries in consideration of MSE and MAE criterion. Similarly, for the first period, the minimum loss model is F1GARCH-BBM (SST), and for the second period, is EGARCH (GED) for the Henry Flub natural gas series inconsideration of M'S E and MAE criterion.

Ham *et al.* (2016) evaluated the comparative performance of volatility models to reveal the effects of the global financial crisis on volatility using daily returns of crude oil prices. According to the sample periods, the results of the models highlight that the APGARCH and FIAPGARCH modelswith Student-t and skewed Student’s t distributions best fit oil prices. Furthermore, when considering the global financial crisis, the results showed that the crude oil price arc is characterized by high volatilities and has long memory effects, as expected.

Ijeoma *et al.* (2016) examined the effect of oil prices on volatility in food prices in Nigeria. This study specifically considered the long-run, short-run, and causal relationships between these variables. Annual data on oil prices and individual prices of maize, rice, sorghum, soya beans, and wheal spanning 2000 to 2003 were used. The price volatility for each crop was obtained using the Generalized Autoregressive Conditional Heteroskedascity (GARCH (1,1) model. The measure of oil price was the Refiner acquisition cost of imported crude oil. The Augmented Dickev-Fuller and Phillip-Perron unit root tests showed that all variables were integrated in order one, I (1). Therefore, the study also used the Johansen co-integration test to examine the long-run relationship. The results show that there is no long-run relationship between oil prices and any individual food price volatility. Thus, this study implemented a VAR model rather than a VECM to investigate the short-run relationship. The VAR model result revealed a positive and significant short-run relationship between oil prices and each of the selected food price volatility, with the exception of rice and wheat price volatility. These results were further confirmed by the impulse response functions. The Granger causality test result indicates a unidirectional causality from oil prices to maize, soya bean, and sorghum price volatilities but did not show such a relationship for rice and wheat price volatilities.

Mahesh et *al.* (2016) analyzed volatility patterns in crude oil price return using both symmetric and asymmetric GARCH family models. The time series data comprise daily spot and near-month expiry futures contract prices of crude oil sourced from MCX for the past 10 years. January 2006 to December 2015. Based on the AIC and SIC1 principles; the study revealed thatGARCH (1,1) and EGARCH (LI) models with student’s distribution were found to better analyze the symmetric and asymmetric volatility estimates of near-month expiry futures contract crude oil price returns.

Bahar *et al.* (2017) used West Texas Intermediate daily data from 2nd January, 1986 to 31s1 August, 2016, where the result showed that the price of crude oil has structural breaks feature. Moreover, the forecasting result showed high accuracy with geometric Brownian motion compared with the mean-reverting Onrstein-Uhlenbeck process for the short term.

"Thomas *et al.* (2016) adopted the Markov-switching multi-fractal (MSM) model and a battery of generalized autoregressive conditional heteroscedasticity (GARCH)-type models to model and forecast volatility in oil prices." Extending previous work by Wei *et al.*, (2010) and Wang *et al.*, (2016), this study evaluated the forecasting performance of all these models via a superior predictive ability (SPA) test. The study also considered volatility in oil prices in the nineteenth century along with recent data, applied different types of MSM models, and considered value-at-risk predictions in addition to forecasting volatility. Confirming its successful performance in other studies, the new MSM model emerged as the model that most often cannot be outperformed by other models across forecasting horizons and subsamples. This superiority was also applied to the value-at-risk forecasting.

Abduchakeem *et al.* (2016) analyzed volatility in oil prices—macroeconomic volatility in Nigeria using the GARCH model and its variants (GARCH-M, EGARCH and TGARCH) using daily, monthly, and quarterly data. From the finds the result revealed that: all the macroeconomic variables considered (real gross domestic product, interest rate, exchange rate and oil price) were highly volatile; the asymmetric models (TGARCH-M suggested that oil price was a major source of economic volatility in Nigeria.

Olugbenga *et al.* (2017) studied the impact of volatility in oil prices on investment decision-making in marginal field development in Nigeria. This study also investigated the relationship between volatility in oil prices and marginal field investment analysis in Nigeria. The marginal field's crude oil production was used as a replacement for the investment analysis. A monthly data from October 2015 to April 2016 were considered. The GARCH model, Johansen co-integration, and Granger causality tests were used to estimate the results. However, the result showed a significant positive relationship between oh! Price volatility and crude oil production ($P < 0.05$)

Deebom, and Isaac, (2017) conducted a study targeted at modeling price volatility and the risk-return related to crude oil export in the Nigerian crude oil market using the first order asymmetric and symmetric univariate Generalized Autoregressive Conditional Heteroscedasticity (GARCH) family model under three distributional assumptions, namely, normal, student* s-t, and generalized error distribution. The data for this study were extracted from the Central Bank of Nigeria online statistical database from January 1987 to June and 2017. The results from the statistical analysis revealed that the markets were optimistic- of their investment and other trade-related activities. In addition, there were higher probabilities of gains than losses. Although the variables used in these markets were extremely volatile, they provide evidence that there exists positive risk-first-rated meaning that investments or investors deserved rewards for holding risky assets. In the estimation, the first-order symmetric GARCH model (GARCH, (1, 1) in Student's t-error assumption gave a better fit than the first-order Asymmetric GARCH model (EGARCH (1, 1)) in Normal error distributional assumptions. However, the selected models were subjected to several diagnostic tests, such as the ARCH wheel test, the test for serial correlation and QQ-plot in order to validate their fitness, which was confirmed to be appropriate. This study recommends that when modeling volatility of price return for certain micro/ macroeconomic variables, the leverage effect of such variables should be properly estimated using an asymmetric GARCH model.

Ayeni, (2018) examined the short- and long-run effects of oil price shocks and exchange rate volatility on investment in Nigeria using annual time series data from 1981 to 2016. The stationarity property of the series was examined using both the Augmented Dickey Fuller (ADF) test and the Phillip Perron (PP) unit tool test, while the Autoregressive Distributed Lag (ARDL) Bounds Cointegration test was employed to examine whether the series were co integrate. The unit root results for both tests were consistent and revealed that the series combined 1(0) and 1(1). The Bounds Co-integration test showed that the variables were co-integrated. Consequently, the short run and a long run ARDL model were estimated. The results show that exchange rate volatility significantly affects investment both in the short and long run, while oil price shock and other variables have insignificant impact.

Onyeka-Ubaka *et al.* (2018) analyzed volatility patterns in crude oil price return using autoregressive integrated moving average (ARIMA) and generalized autoregressive conditional heteroskedasticity (GARCH) family models. The results revealed that the GARCH (1, 1) and ARIMA (1, I, 0) models performed well in capturing the stylistic features present in high-frequency crude oil prices in Nigeria during the sample period. The Ho It-Winters forecast made for twenty-six (26) months using ARIMA ('1, k 0) was approximately close to the real price of crude oil per barrel, as evident from the 95% confidence interval estimates.

Christopher N. E et al. (2018) estimated the optimal forecasting model for stock returns and the nature of stock returns volatility in Nigeria using daily All-Share stock data. This study estimatessix sets of symmetric and asymmetric GARCH-family models of stock returns volatility (three of which are augmented with trading volume) in three different sets of error distributions: normal student's t and generalized error distribution (GED) with a view to selecting the model with the best predictive power. Relying on root mean square error (RMSE) and Thiel's Inequality Coefficient, GARCH (1, 1) and augmented EG AR CH (1,1) in GED proved to possess the best forecasting capability as adjudged by the last 30 days out-of-sample forecast. This study revealed a leverage effect and a decline in the persistence parameter after incorporating trading volume. Overall, the result provided evidence of a high probability of negative return from investment in the Nigerian stock market durin GARCH g the sample period. The empirical merit of the model was its potential for applications in the analysis of value at risk (VaR) of quoted stocks and, therefore, the evaluation of risk premia that guide investors' stock portfolio choices.

Bashir (2018) investigated whether a GARCH-family model may be relevant in forecasting Nigerian crude oil prices. This study considered monthly bonnylight crude oil price data from April 1986 to December 2015. The statistical properties of the series were investigated using ADR BP, and KPSS. The unit root test results indicated that the MBLP was n on-stationary at level but stationary at first difference. The study also found that the GARCH family models could capture the high and persistent volatility of the bonny fight oil price. Moreover, among the GARCH- family models, the symmetric GARCH (E ERGED model was found to be the parsimonious model and to perform better forecasting than other GARCH family models. Out-of-sample forecast periods covering the periods of January 2016 to December 2016 showed that bonny light crude oil prices hovered around $25.8 to $55.8.

Sujoy and Arshad (2018) analyzed Various GARCH family models to forecast volatility and output in terms of return vectors. These models are used as inputs for a neural network. The return forecasting performance of the GARCH family models was compared with GARCH- ANN models using root mean square error as the criteria. The results show that the hybrid ANN–EGARC11 model gives the best performance. An explanatory variable, the exchange rate between the Indian rupee and the Saudi Arabian riyal, was used as input for the neural network model for the second scenario and using the same criteria of root mean square error. It was observed to have no improvement over the previous ANN-GARCH models.

Died and Predj (2019) hocused on price volatility and uncertainty over the period January 1986-December 2018, covering episodes of oil price increases and collapses. Accordingly, in line with Poon and Granger (2003) and Terasvirta and Zhao (2011), the study proposed three different specifications of stochastic oil volatility: standard stochastic volatility, stochastic volatility moving average, and leverage stochastic volatility models. Computation of out-of- sample forecasts for uncertainty in oil prices using estimates for these three stochastic oil price volatility models was performed. The results show that the standard stochastic volatility model outperforms other Iwo models when focusing on oil price uncertainty. These findings are relevant to better forecast and understand the effects of oil price uncertainty on the real economy.

Awidan (2019) analyzed the behavior of crude oil prices' behavior and determined the dynamic relationships between domestic crude oil prices and fundamental macroeconomic variables in Libya and Nigeria. The analysis in this study involved two stages. The first stage was to analyze and model oil price returns of the Libyan, Nigerian and OPEC markets; the study also examined the existence of a structural break in crude oil price data. The empirical analysis used the AR-GARCH, AR-EGARCH, AR-GJR-GARCM, AR-APARCH, AR-CGARCH, and AR-ACGARCH models to model the conditional mean and conditional variance of the oil price return sunder throe error distributions, namely the normal distribution, Student's distribution, and generalized error distribution. The results show that the three-return series exhibits no structural break in the mean and variance equations, but the study finds evidence of volatility1 clustering and leverage effect response to good and bad news in the asymmetric models for the three markets. The study also assessed out-of-sample forecasts of the class of GARCH models using four loss functions. The results indicate that the AR-CGARCH-GED model is the best model for forecasting oil returns in Libya, whereas the best models for Nigeria and the OPEC arc the AR-GARC1LGED and AR-EGARCH-t models, respectively. The second stage examines the dynamic relationship between oil prices and GDP, exchange rate, and inflation using annual data for the 1970-2017 periods in Libya and Nigeria. Both short-run and long-run relationships between these variables were explored by applying co-integration tests, the vector autoregressive model (VAR), and vector error correction (VECM) model. Granger causality tests, impulse response Junctions, and forecast variance decompositions. The results showed that there was a co-integrating relationship between domestic oil prices and macroeconomic variables are co integrating in Libya and Nigeria. Furthermore, the results showed that there was a unidirectional Granger-causality relationship running from Libyan oil prices to Libya's GDP. Moreover, the results showed a unidirectional causality running from Nigerian oil prices to GDP and the exchange rate in Nigeria. The findings of the impulse response functions suggest significant impacts of domestic oil price shocks on macroeconomic variables in Libya and Nigeria in the short and long term. The results of the variance decomposition analysis indicate that changes in Libyan oil prices can impact Libyan GDP. Nigerian oil price shocks affect most macroeconomic variables in Nigeria.

Fazelabdolabadi (2019) proposed the forecasting of the crude oil prices using a hybrid Bayesian Network (BN) method. The results demonstrate that the proposed method is a good choice for short-term forecasting.

Yue-Jun. *et al.* (2019) estimated and forecasted volatility in crude oil prices using three single- regime GARCH (he... GARCH, GJR-GARCH and EGARCH) and two regime-switching GARCH (he, MMGARCH and MRS-GARCH) models. The model confidence set (MCS) procedure was also employed to evaluate forecasting performance. The in-sample results show that the MRS-GARCH model provided higher estimation accuracy for weekly data. However, the out-of-sample results showed the limited significance of considering regime switching. Overall, the results indicated that the incorporation of regime switching did not perform significantly better than the single-regime GARCH models. The findings proved robust to both daily and weekly data for WT1 and Brent over different time horizons.

Kuhe (2019) investigated the dynamic relationship between crude oil prices and stock market price volatility in Nigeria using the cointegrated vector generalized autoregressive conditional heteroskedasticity (VAR-GARCH) model. The study used monthly data on the study variables from January 2006 to April 2017 and employed Dickey-Fuller Generalized least squares unit root test, simple linear regression model, and unrestricted vector autoregressive model. The Granger causality test and standard GARCH model as methods of analysis. Results showed that the study variables were integrated of order one, and no long-run stable relationship was found to exist between crude oil prices and stock market prices in Nigeria. Both crude oil prices and stock market prices were found to have positive and significant impacts on each other, indicating that an increase in crude oil prices will increase stock market prices and vice versa. Both crude oil and stock market prices were found to have predictive information on one another in the long- run. A one-way causality ran from crude oil prices to stock market prices, suggesting that crude oil prices determine stock prices and are a driving force in the Nigerian stock market. Results of GARCH (1, 1) models show high persistence of shocks in the conditional variance of both returns. The conditional volatility of stock market price log return was found to be stable and predictable, while that of crude oil puce log return was found to be unstable and unpredictable, although a dependable and dynamic relationship between crude oil prices and stock market prices was found to exist.

Lu-Tao *et al.* (2019) used various fractional GARCH models to describe typical volatility characteristics such as long memory, volatility clustering, asymmetry, and a thick tail. The autoregressive conditional heteroscedasticity in the mean model (ARCH-M) and the peak-over-threshold model of extreme value theory (EVT-POT) were taken into account to develop a hybrid time-varying long memory GAROTM-EVT model for calculation of static and dynamic VaR. The empirical results showed that the WT1 crude oil has a significantly long memory feature. All fractional integration GARCHmodels could describe long memory appropriately, and the Fl APARCH model was the best in regression and out-of-sample one-step-ahead VaR forecasting. Back-testing results showed that the FIAPARCH-M-EVT model was superior to other GARCH--type models that only- consider oil price fluctuation characteristics partially and traditional methods including Variance-Covariance and Monte Carlo in price risk measurement. Conclusions confirmed that considering long memory, asymmetry, and fat tails in the behavior of energy commodity return combined with effectively dynamic time-varying risk reflection such as the ARCH-M model and reliable tail extreme filter processes such as EVT could improve the accuracy of crude oil price risk measurement, provide an effective tool for analyzing the extreme risk of the tail of the oil market, and facilitate risk management for oil market investors.

Amare *et al.* (2020) modeled and forecasted the silver price volatility dynamics on the Ethiopian market using GARCH family models using data from January 1998 to January 2014. The price return series for silver shows the characteristics of financial time series, such as leptokurtic distributions, and thus could suitably be modeled using GARCH family models. An empirical investigation was conducted on model price volatility using GARCH family models. Among the GARCH family models considered in the study, the ARMA (1, 3)-EGARCH (3, 2) model with the normal distributional assumption of residuals was found to be a better fit for the price volatility of silver. Among the exogenous variables considered in this study, the saving interest rate and general inflation rate has a statistically significant effect on monthly volatility in silver prices. In the EGARCH (3, 2) volatility model, the asymmetric term was found to be positive and significant. This was an indication that the unanticipated price increase had a greater impact on price volatility than unanticipated price decreases for silver.

Boitumelo, Yolanda, S; Johannes, T. T; Lebotsa, D; M. (2020). The ARCH, GARCH, and EGARCH models were used to model volatility in oil prices and macroeconomic variables in South Africa for the period 1990Q1 to 2018Q2. The macroeconomic variables used in this study wereGDP, inflation, interest rates, and exchange rates. According to ARCH (1) and GARCH (1, 1) models, the exchange rate and interest rate have a negative effect on oil prices, whereas GDP andinflation have a positive effect. The results for GDP and inflation implied that a 1%increase in GDP and inflation may increase oil prices. The negative effect on interestrate and exchange rates, as indicated by their negative values, implies that a 1% increase in the interest and exchange rates may lead to a decrease in oil prices. The EGARCH (1,1) model reveals that oilprices are negatively affected by all macroeconomic variables. This implies that a 1% increase in these variables could decrease oil prices. The symmetric and asymmetric techniques revealed that South African oil prices were volatile.

Yu and Fixing (2020) compared uni-regime GARCH-type models and GARCH-type models with Markov and hidden Markov (1-l. M) switching regimes on their forecasting abilities in WTI and Daqing crude oil markets, respectively. The empirical results indicated that the HM-EGARCH model outperformed the competitive models, namely, the regular GARCH-type models and Markov regime-switching models, as well as the other models with hidden Markov regimes, through the results of six loss functions and the superior predictive ability (SPA) test. More significantly, the result showed that HM-EGARCH did not only perform well in developed but also emerging crude oil markets. Therefore, the HM-EGARCH model can be regarded as an effective measure of volatility when accounting for different volatility states in a time-changing process.

Ngonzi *et al.* (2020) investigated volatility in daily stock returns for Total Nigeria Plc using nine variants of GARCH models: sGARCH, girGARCH, eGARCH, iGARCH, aGARCH, TGARCH, NGARCH, NAGARC11, and AVGARCH, along with value at risk estimation and back testing. Daily data for Total Nigeria Plc returns for the period January 2, 2001 to May 8. 2017, and concluded that eGARCH and sGARCH performed better for normal innovations, while NGARCH performed better for student t innovations. This investigation of the volatility, VaR. and back-testing of the daily stock price of Total Nigeria Plc is important because most previous studies covering the Nigerian stock market have not paid much attention to the application of back-testing as a primary approach. The results of the estimations revealed that the persistence of the GARCH. models were stable except for few cases for which iGARCH and eGARCH were unstable. Additionally, for student t innovation, the sGARCH and girGARCH models failed to converge; the mean reverting number of days for returns differed between models.

Geleta *et al.* (2020) proposed an innovative semi-parametric nonlinear fuzzy-EG ARCH-ANN model to solve the problem of accurate modeling for forecasting stock market volatility. The model was developed by a combination of the FIS, ANN. and EGARCH models. Because the proposed model was highly nonlinear and gradient-based parameter estimation methods might not give global optimal parameters for highly nonlinear models, the study decided to use evolutionary algorithms instead. In particular, a differential evolution (DE) algorithm was proposed to solve the parameter estimation problem of the proposed model. Subsequently, the semiparametric nonlinear" fuzzy-EG ARCH-ANN model was developed mathematically from the three models mentioned above, and the study simulated data using it. After the simulation, parameter estimation of the proposed model was performed using a differential evolution algorithm on the simulated data. Finally, the proposed model was good in capturing the volatility clustering and leverage effects of highly nonlinear and complicated financial time-series data that were overlooked by the EGARCH model.

Dum *et al.* (2021) conducted a study aimed at developing an appropriate GARCH model for modeling crude oil prices in Nigeria using symmetric and Asymmetric GARCH models. The specific objectives of the study were:

to build an appropriate Symmetric and asymmetric Generalized Autoregressive Conditional Heteroskedacity (GARCH) model for Nigerian crude oil prices and compare the advantages of using Symmetric and Asymmetric GARCH. The data for this study were extracted from the Central Bank of Nigeria online statistical database from January 1982 to December. 2018. Two classes of models were used in the study; they were symmetric and Asymmetric GARCH models. The results of the estimated models revealed that asymmetric GARCH. Model (EGARCH (1,1) in student's-! error assumption gave a better fit than the first-order Symmetric GARCH models. Additionally, using EGARCH (1,1) models with their corresponding error distribution in estimating crude oil prices, it was found that the larger the size of the estimated news components of the model, the higher the negative news associated with a high impact of volatility. This means that conditional volatility estimated using the EGARCH model has strong asymmetric characteristics that are prone to news sensitivity.

RESEARCH METHODOLOGY

Research Design

In this section, we present the data source, data transformation, and data analysis methods. This section specifically hinges on the source of data and data transformation, and preliminary tests, such as descriptive statistics and normality measures, time plots, the unit root test, the Portmanteau test, and the Hetcroskedasticity test. The section presents mode) specifications, model order selection, estimation procedure, diagnostic checks, and forecasting and evaluation.

Source of Data and Data Transformation

The data used in this study are the secondary monthly and daily time-series data on crude oil prices in Nigeria from January 2006 to September 2022 and from November 3rd, 2009 to November 4th, 2022, obtained from www.cbn.ng.org. Crude oil prices Pt are converted to log-return series rr using the following equation:

$$r_t = 100.\ln \nabla P_t \quad (3.1)$$

where; $\nabla P_t = \ln(P_t - P_{t-1})$, r_t denotes the log return series, and Pt denotes the closing crude oil price index at the current month t.

Methods of Data Analysis

For data analysis in this work, the following statistical tools were used.

Descriptive Statistics and Normality Measures

Descriptive statistics, such as arithmetic mean and standard deviation, as well as normality measures, were employed to summarize the characteristics of crude oil prices and returns. The mean of any given dataset is computed as follows:

$$\bar{r} = \frac{1}{n}\sum_{i=1}^{n} r_t \quad (3\text{-}2)$$

The sample standard deviation of any given set of data over a given period is computed using the following formula:

$$\hat{\sigma} = \sqrt{\frac{1}{n-1}\sum_{t=1}^{n}(r_t - \bar{r})^2} \quad (3.3)$$

where; $\bar{r}$ is the sample mean, n is the sample size.

A normality test, which is also a goodness-of Tit test of whether sample data have the skewness and kurtosis that match a normal distribution, was proposed by Jarquc and Bera (1980, 1987), who called the Jarque Bera test of normality. The Jarque-Bera (JB) test tests the null hypothesis that a given dataset is normally distributed. Given a series (r£), the JB test statistic is defined as follows:

$$JB = \frac{T}{6}\left(g_1^2 + \frac{1}{4}(g_2 - 3)^2\right) \quad (3.4)$$

where; g_1 is the sample kurtosis defined as follows:

$$g_1 = \frac{\mu_3}{\mu_2^{2/3}} = T^{1/2} \sum_{t=1}^{T} (r_t - \bar{r})^3 \bigg/ \left(\sum_{t=1}^{T} (r_t - \bar{r})^2 \right)^{3/2} \quad (3.5)$$

and g_2 is the sample kurtosis, which is defined as

$$g_2 = \frac{\mu_4}{\mu_2^2} = T \sum_{t=1}^{T} (r_t - \bar{r})^4 \bigg/ \left(\sum_{t=1}^{T} (r_t - \bar{r})^2 \right)^2 \quad (3.6)$$

where T is the number of observations and $\bar{r}$ is the sample mean. The normal distribution has askewness equal to 0 with kurtosis of 3. The JB normality test checks the following pair of hypotheses:
H_0: $\hat{\mu}_3 = 0$(i.e.r_t is normally distributed) against the alternative
H_0: $\hat{\mu}_3 \neq 0$ and Ho: g3 =A 0 and $\hat{\mu}_4 \neq 0$ (i.e., $r_t$is not normally distributed). The test rejects the null hypothesis if the p-value of the JB test statistic is lessthan a= 0.05 level of significance.

Time Series Plots of Level and Transformed Series

By first plotting the series against lime, we can assess the trend movement die pattern and determine whether any structural breaks, outliers, or data errors occur. This step may also reveal whether a significant seasonal pattern exists in the time series.

Dickey-Fuller Generalized Least Squares (DF GLS) Unit Root Test

We employ the Dickey-fuller Generalized Least Squares (DF GLS) unit root test to investigate the unit root property and order of integration of oil prices and returns in Nigeria. The DFGLS test involves estimating the standard ADF test equation as follows:

$\Delta r_t = \alpha r_{t-1} + X_t'\delta + \beta_1 \Delta r_{t-1} + \beta_2 \Delta r_{t-2} + \cdots + \beta_p \Delta r_{t-p} + \varepsilon_t$(3.7)

After substituting the DFGLS detrended $r_t^d$for the original r_t, we have

$\Delta r_t^d = \alpha r_{t-1}^d + \beta_1 \Delta r_{t-1}^d + \cdots + \beta_p \Delta r_{t-p}^d + \varepsilon_t$ (3.8)

As in the ADF test, we consider the t-ratio for $\hat{\alpha}$from this test equation and evaluate

$t_\alpha = \frac{\hat{\alpha}}{(se(\hat{\alpha}))}$(3.9)

where $\hat{\alpha}$ is the estimate of α, and$se(\hat{\alpha})$ is the coefficient standard error. The null and alternative hypotheses are written asH_0: $\alpha = 0$ againstH_1: $\alpha < 0$. The test rejects the null hypothesis of unit root if the DFGLS test statistic is less than the test critical values at the designated test sizes (Elliot *et al*., 1996).

Hetcroskedasdeity Test

To test for heteroskedasticity or the ARCH effect in the residuals of returns, we apply the Lagrange Multiplier (LM) test as proposed by Engle (1982). The procedure of performing the Engle's LM test is to first obtain the residuals et from an ordinary least squares regression of the conditional mean equation, which could be an AR, MA, or ARMA model that best fit the data. For instance, in the ARMA (1,1) model, the conditional mean equation is specified as

$r_t = \alpha_1 r_{t-1} + \varepsilon_t + \beta_1 \varepsilon_{t-1}$ (3.10)

where rt is the return series, , $\alpha_1$and β_1 are the coefficients of the AR and MA terms, and et is the random error term. Having obtained the residuals e£, we then regress the squared residuals on a constant and q lags such as in the following equation:

$e_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \alpha_2 e_{t-2}^2 + \alpha_3 e_{t-3}^2 + \cdots + \alpha_q e_{t-q}^2 + v_t$ (3.11)

The null hypothesis of no ARCH effect up to lag q is then expressed as follows:

$H_0: \alpha_1 = \alpha_2 = \alpha_3 = \cdots = \alpha_q$ Versus the alternative $H_1: \alpha_i > 0$ for at least one $i = 1,2,3, \ldots, q$.

There are two test statistics for the joint significance of the q-laggedsquared residuals. The 1-statistic and lire number of observations times R-squared (nR^2) from the regression. The F- statistic is estimated as follows:

$$F = \frac{SSR_0 - SSR_1/q}{SSR_1(n - 2q - 1)} \quad (3.12)$$

$$\text{where } SSR_1 = \sum_{t=q+1}^{T} e_t^2, SSR_0 = \sum_{t=q+1}^{T} (r_t^2 - \bar{r})^2 \text{ and } \bar{r} = \frac{1}{n}\sum_{t=1}^{T} r_t^2 \quad (3.13)$$

$\hat{e}_t$ is the residual obtained from least squares linear regression, $\bar{r}$ is the sample mean of r_t^2. The nR^2 is evaluated against $\chi^2(q)$ distribution with q degrees of freedom under H_0. The decision is to reject the null hypothesis of no ARCH effect in the residuals of returns is rejected if the p-values of the F-statistic and nR^2 statistic are less than $\alpha = 0.05$.

Model Specifications

To specify an autoregressive moving average (ARMA) cum generalized autoregressive conditional heteroskedasticity (GARCH) process, we start with an autoregressive (AR) process, a moving average (MA) process, an autoregressive moving average (ARMA) process, an autoregressive conditional heteroskedasticity (ARCH) process, and a generalized autoregressive conditional heteroskedasticity (GARCH) process, which are specified in the following subsections.

Autoregressive (AR) model

According to Box and Jenkins (1970), an autoregressive model of order p, denoted by AR (p), is given by the following expression:

$Y_t = \alpha_0 + \alpha_1 r_{t-1} + \alpha_2 r_{t-2} + \cdots + \alpha_p r_{t-p} + \varepsilon_t$ (3.14)

where r_t is the return series at time$t$, ε_t is a purely random process with mean zero and variance σ^2, α_0 is a constant and $\alpha_1, \alpha_2, \ldots, \alpha_p$are autoregressive parameters, and the subscripts are the orders of the autoregressive parameters that increase with increases in r_t. The values of α_i which would make the process to be stationary are such that the roots of the polynomial equation $\Phi[L] = 0$ lie outside the unit circle in the complex plane. Here, L is the lag operator such that$L^j r_t = r_{t-j}$.

Moving Average (MA) Process

Suppose that $\{\varepsilon_t\}$ is a white-noise process with mean zero and varianceσ^2, then the process r_t is said to be a moving-average model of order q, denoted as MA (q) if

$r_t = \varepsilon_t + \beta_1 \varepsilon_{t-1} + \beta_2 \varepsilon_{t-2} + \cdots + \beta_q \varepsilon_{t-q} = \varepsilon_t + \sum_{i=1}^{q} \beta_i \varepsilon_{t-i}$ (3.15)

Where pi is the moving average parameter. The subscript on p^ are called the orders of the moving average parameters.

Autoregressive Moving Average (ARMA) Process

A stochastic process resulting from a combination of autoregressive and moving average models is called an Autoregressive Moving Average (ARMA) model. An ARMA model of order p, q written as ARMA (p,q) is specified as

$r_t = \alpha_1 r_{t-1} + \alpha_2 r_{t-2} + \cdots + \alpha_p r_{t-p} + \varepsilon_t + \beta_1 \varepsilon_{t-1} + \beta_2 \varepsilon_{t-2} + \cdots + \beta_q \varepsilon_{t-q}$ (3.16)

where Qarc represents the autoregressive parameters, ft represents the moving average parameters, and p and q represent the orders of the autoregressive and moving average parameters,, respectively.

Autoregressive Conditional Heteroskedasticity (ARCH) Model

The autoregressive conditional heteroskedasticity model of order q, ARCH (q) proposed by Engle (1982) is given by the following:

$$r_t = \mu_t + \varepsilon_t \text{ (3.17)}$$

$$\varepsilon_t = \sigma_t e_t;\ e_t \sim N(0,1) \text{ (3.18)}$$

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \cdots + \alpha_q \varepsilon_{t-q}^2 = \omega + \sum_{i=1}^{q} \alpha_i\, \varepsilon_{t-i}^2 \text{ (3.19)}$$

where ε_t is the shock at day t and it follows heteroskedastic error process, σ_t^2 is the volatility at day t, $\varepsilon_{t-i}^2$is the squared innovation at day $t - i$ andω is a constant term.

The Generalized ARCH (GARCH) Model

The ARCH model was generalized to the GARCH model by Bollerslev (1986). A Generalized Autoregressive Conditional Heteroskcdastiqjty process is considered a GARCH (p, q) process.

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{q} \beta_i \sigma_{t-i}^2 \text{ (3.20)}$$

where ε_t^2 is the ARCH term σ_t^2 is the GARCH term. The above model is stationary for variance and covariance if the following necessary conditions are satisfied: $\omega > 0$; $\alpha_i > 0, i = 1, 2, \ldots, q$; $\beta_i > 0, i = 1, 2, \ldots, p$ and $\sum \alpha_i + \sum \beta_i < 1$. this summation indicates the persistence of a volatility shock. Bollerslev, Chou, and Kroner (1992) showed that basic GARCH (1,1) model is sufficient for capturing all volatility in any financial time series. The basic GARCH (1,1) is expressed as follows:

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \qquad (3.21)$$

In many empirical applications using high-frequency financial time-series data, extreme persistence in conditional variance can be observed; thus, in the basic GARCH (1,1) model, the sum of ARCH and GARCH parameters is close to unity.

Threshold GARCH (TGARCH) Model

The Threshold GARCH (TARCH) mode! Was introduced independently by Glosten*et at* (1993) and Zakoi an (1994). This model al RHYS for asymmetric shocks to volatility. The conditional variance of the simple TARCH (1,1) mode1 is defined as follows:

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \gamma \varepsilon_{t-1}^2 d_{t-1} \qquad (3.22)$$

Where $d_t = 1$ if ε_t is negative and 0 otherwise. In the TGARCH (1,1) model, volatility tends to increase with bad news $(\varepsilon_{t-1} < 0)$ and decreases with good news $(\varepsilon_{t-1} > 0)$. Good news impacts$\alpha_1$whereas bad news impacts$\alpha_1 + \gamma$. If the leverage effect parameter $\gamma > 0$ and statistically significant, then the leverage effect exists. If $\gamma \neq 0$, the shock is asymmetric, and if $\gamma = 0$, the shock is symmetric. The persistence of shocks in volatility is measured by $\alpha_1 + \beta_1 + \gamma/2$.

Model Selection Criteria

To select the best-fitting ARMA-GARCH model, Akaike Information Criteria (AIC) based on (Akaike, 1974), Schwarz Information Criterion (SIC) based on (Schwarz, 1978) and Hannan-Quinn Information Criterion (HQC) based on (Hannan, 1980), and log likelihood are the most commonly used model selection criteria. These criteria are used in this study and are computed as follows:

AIC(K) = -2 log L +2*K* (3.23)

SIC(K)= -21ogL + *K*logT (3.24)

HQC(*K*) = 2 log[logT] *K*2 log L. (3.25)

where *K* is the number of independently estimated parameters in the model, T is the number of observations: L, is the maximized value of the log li GARCH kelihood for the estimated model defined as follows:

$$L = \prod_{i=0}^{n} \left(\frac{1}{2\pi\sigma_i^2}\right)^{1/2} exp\left[-\sum_{i=1}^{n} \frac{(y_i - f(x))^2}{2\sigma_i^2}\right] \quad 3.26$$

$$\ln(L) = In\left[\prod_{i=1}^{n}\left(\frac{1}{2\pi\sigma_i^2}\right)^{1/2}\right] - \frac{1}{2}\sum_{i=1}^{n}\frac{\left(y_i - f(x)\right)^2}{\sigma_i^2} \qquad (3.27)$$

Thus, given a set of estimated ARMA-GARCH models for a given set of data, the preferred model is the one with the minimum information criteria and larger log likelihood value.

Estimation of ARMA-GARCH Models and Error Distributions

When modeling returns series for high-frequency time-frequency data, the estimates of ARMA-
The GARCH process is obtained by maximizing the following likelihood function:

$$L\theta_t = -\,{}^{1}\!/_{2}\sum_{t-1}^{T}\left(\ln 2\pi + ln\sigma_t^2 + \frac{\varepsilon_t^2}{\sigma_t^2}\right) \qquad (3.28)$$

The three error distributions are defined as follows:

(l)The normal (Gaussian) distribution is given by

$$f(z) = \frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}, -\infty < z < \infty \qquad (3.29)$$

(2) The Student-t distribution is defined as follows:

$$f(z) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{v\pi}\,\Gamma\left(\frac{v}{2}\right)}\left(1 + \frac{z^2}{v}\right)^{-\left(\frac{v+1}{2}\right)}, -\infty < z < \infty\,. \qquad (3.30)$$

where *denotes* the number of degrees of freedom, and T denotes the Gamma function. The degree of freedom *v*> 2 controls the tail behavior. The *t-distribution* approaches the normal
Distribution as $v \to \infty$.

(3) The Generalized Error Distribution (GED) is given as

$$f(z,\mu,\sigma,v) = \frac{\sigma^{-1}ve^{\left(-\frac{1}{2}\left|\frac{\left(\frac{z-\mu}{\sigma}\right)}{\lambda}\right|^{v}\right)}}{\lambda 2^{(1+(1/v))}\Gamma\left(\frac{1}{v}\right)}, 1 < z < \infty \ (3.31)$$

v> 0 is the degrees of freedom or tail-thickness parameter, and

$$\lambda = \sqrt{2^{(-2/v)}\Gamma\left(\frac{1}{v}\right)\Big/\Gamma\left(\frac{3}{v}\right)}$$

If *v* = 2, the GED yields a normal distribution. If *v*< 1, the density function has thick tails as follows:
Than the normal density function, whereas for *v*> 2, it has thinner tails.

Model Diagnostic Checking

When a time series model, such as GARCH models, has been fitted to a given dataset, it is advisable to check that the model documents provide an adequate description of the data. In doing so, we employed the Lagrange Multiplier Engle Heteroskedasiicity test for ARCH effects earlier discussed in section 3.2.4.

Justification of Methods

1. To test for stationary, we used the Augmented Dickey Fuller test in conjunction with the KPSS test because they are popular tests that check for trend stationary data.
2. To test for Hereoskedasticity. We use the Lagrange Multiplier (LM)g test proposed by Engle (1982).It is because it regresses the square error of its lag.
3. We compare the best fitting model for forecasting because the model that will be fitted will be useful to individuals, stakeholders, policymakers, and the government for planning and predictions, thus enabling them to prepare well in advance for future occurrence. Also, it serves as a reference document for research purposes for statisticians.

Result

Summary Statistics and Normality Measures of Crude Oil Prices and Returns

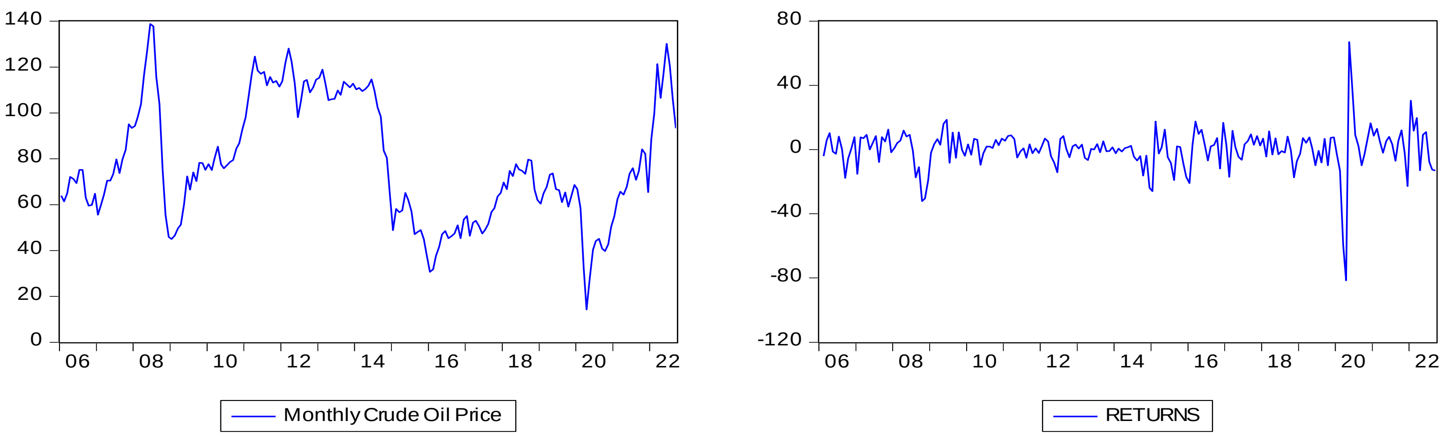
To understand the descriptive and distributional characteristics of crude oil prices and returns, summary statistics, such as monthly and daily means, median, maximum, and minimum prices and returns, standard deviation^ as well as normality measures, such as skewness, kurtosis and Jarque-Bera statistics, for both monthly and daily crude oil prices and returns are computed and presented in Table 1.

Graphical Examination of Crude Oil Prices and Return Series

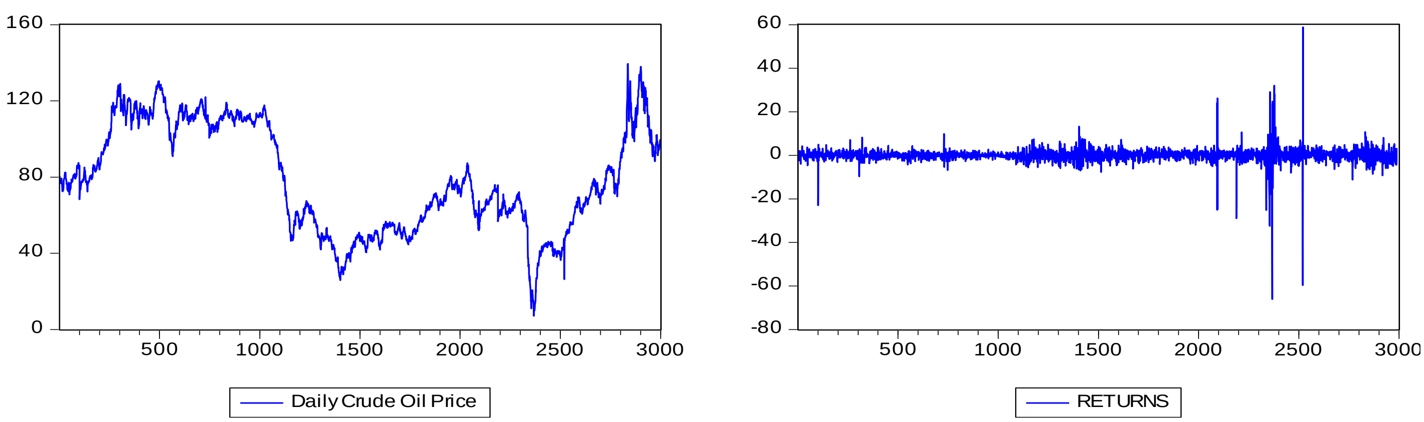
The first step in analyzing time series data is to plot the original series in level and first difference against time and observe its graphical properties. This helps in understanding the trends and patterns of movement of the original and transformed series. Here, we plot the original crude oil price series and the returns against time and observe the features of the data. The time series plots are presented in Figure 1.

Table 1: Summary7 Statistics of Crude Oil Prices and Returns

Variable	Monthly Daily	Prices Returns Prices Returns		
Mean	78.1723	0.1894	78.6217	0.0072
Median	73.6532	1.3894	74.2536	0.0591
Maximum	138.7400	66.9767	139.413	58.8928
Minimum	14,2800	-81.5898	7.15000	-6.0451
Standard Dev.	26.5285	12.9212	27.9037	3.5176
Skewness	0.260Sh	-1.1699	0.0854	-2.0811
Kurtosis	2.0874	15.4473	1.8616	114.1962
Jarque-Bera	9.2546	1336.75	165.64	1547224
P-value	0.0099	0.00000	0.00000	0.00000
No. of Obs.	201	- 200	3000	2999



(a) Monthly crude oil prices and returns



(a) Daily crude oil prices and returns

Figure1: Time Plots of Crude Oil Prices and Returns

Unit Root and Heteroskedasticily Tests of Returns

To test for unit roof in the monthly and daily crude oil prices and returns, Dickey-Fuller Generalized Least Squares Elliott, Rothenberg, and Stock (DF GLS (ERS)) unit root test was employed, and the results are presented in the upper panel of Table 2. Furthermore, Engle's Lagrange Multiplier (LM) test for the ARCH effect was employed to investigate heteroskedasticity in the residuals of the monthly and daily crude oil prices and returns series, and the results are presented in the lower panel of Table 2.

Table 2: Unit Root and Test Results and Hcteroskedascity Test for ARCH Effects

DF GLS (ERS) Unit Root and Test				
Variable	**Option**	**DF GLS Test** Statistic value	**1 % Critical**	**5% critical**
Monthly Oil	Intercept only	-1.4969	-2.5766	-1.9424
Price	Intercept and trend	-1.6126	-3.4612	-2.9310
Monthly Oil	Intercept only	-10.6912	-2.5766	-1.9424
Returns	Intercept and trend	-11.1231	-3.4612	-2.9310
Daily Oil	Intercept only	-1.5931	-2.5966	-1.9409
Price	Intercept and trend	-1.579	-3.4612	-2.8900
Daily Oil	Intercept only	-4.1987	-2.5966	-1.9409
Returns	Intercept and trend	-7.1429	-3.4612	-2.8900
Hetcroskcdaseity Pest forARCH Effects				
Variable	F-Statistic	P-value	nR^2	P-value
Monthly Crude Oil Returns	87.05500,	0.0000	59.11943	0.0000
Daily Crude Oil Returns	193.2009	0.0000	180.8883,	0.0000

Model Order Selection and Error Distribution

To select an optimum model order and an appropriate error distribution to model both the monthlyand daily crude oil returns, the Akanke Information Criterion (A1C), Schwartz Information Criterion (SIC) and Hanan-Quinn Criterion (HQC), in conjunction with log likelihood, are considered. This study considers both the lower and upper symmetric and asymmetric GARCH models to model volatility in monthly and daily crude oil returns due to their capability for capturing volatilities in the return series. The model with the least information criteria is considered to be the best fitting model for the data. The model order and error distribution selection results for the monthly and daily crude oil returns are presented in Tables 3 and 4, respectively.

Table 3: Model Order Selection for Symmetric and Asymmetric GARCH Models (Monthly Crude Oil Returns)

Distribution	Model	LogL	AIC	SIC	HQC
Symmetric ARMA-GARCH Model					
Norma	ARMA (1,1)-GARCH (1,1)	-727.38	7.3706	7.4699	7.4108
	ARMA(1,1)-GARCH(I ,2)	-744.85	7.5663	7.6721	7.6032
	ARMA (1,1)-GARCH (2,1)	-725.34	7.3602	7.4760	7.4071
	ARMA (1,1)-GARCH (2,2)	-746.81	7.5860	7.7184	7.6396
SudenGs-t	ARMA (1,1)-GARCH (1,1)	-722.84	7.3375	7.4698	7.3919
	ARMA(1,1)-GARCH (1,2)	-734.74	7.4647	7.5971	7.5183
	ARMA(1,1)-GARCH(2,1)	-724,62	7.3630	7.4954	7.4166
	ARMA (1,1)-GARCH (2,2)	-731.21	7.4393	7.5883	7.4996
GED	ARMA (1,1)-GARCH (1,1)	-723.93	7,3459	7.4618	7.3929
	ARMA (1J j-GARCH(1,2)	-735,90	7.4764	7.6088	7.5299
	ARMA (1,1)-GARCH (2,1)*	-722.05	7.3371	7.4695	7.3907
	ARMA (1,1)-H3ARCH (2,2)	-740.28	7.5304	7.6794	7.5907
Asymmetric ARMA-TARCH Model					

Normal	ARMA(ED-TARCH(1,1)	-721.78	7.3244	7,4403	7.3713
	ARMA(I,1)-TARCH(1,2)	-737.19	7.4893	7.6217	7.5429
	ARMAGD-TARCH (2,1)	-732.52	7.4424	7.5748	7.4960
	ÄRMA(1,1)-TARCH(2,2)	-743.50	7.5628	7.7117	7.6231
Student's-t	ARMA (1,1)-TARCH (1,1)	-720.01	7.3167	7.4491	7.3703
	ARMA (1,1)-TARCH (1,2)	-733.36	7,4609	7,6099	7.5212
	ARMA (1,1)-TARCH. (2,1)*	-716.19	7.2883	7.4373	7.3486
	ARMA(1,1)-TAUCH (2.2)	-731,91	7.4563	7.6218	7.5233
GED	ARMA(1,1)-TARCH(1,1)	-720.48	7.3214	7.4538	7.3750)
	ARMA (ED'TARCH (1,2)	-740.27	7.5304	7.6793	7.5907
	ARMA (IM)-TARCIT (2,1)	-722.52	7.3519	7.5009	7.4123
	ARMA (ED-TARCH(2,2)	-732.81	7.3781	7.6012	7.4462

Table 4: Symmetric and Asymmetric G ARCH Model Order Selection (Daily Crude Oil Returns)

Distribution	Model	LogL	A1C	SIC	HQC
Symmetric GARCH Model					
Normal	GARCH (1;1)	-6818,61	4.5499	4.5579	4.5528
	GARCH (1,2)	-6889.63	4.5980	4.6080	4.6016
	GARCH (2,1)	-6790.40	4.5318	4.5418	4.5354
	GARCH (2,2)	-6764.68	4.5153	4.5273	4.5196
Student's-t	GARCH (1,1)*	-6287.17	4.1962	4.2062	4.1998
	GARCH (1,2)	-6707.88	4.4774	4.4894	4.4817
	GARCH (2,l)	-6287.11	4.1968	4.2088	4.2011
	GARCH (2,2)	-6882.90	4.5948	4.6088	4.5998
GED	GARCH (1,1)	-6351.03	4.2388	4.2488	4.2424
	GARCH (1,2)	-6350.98	4.2394	4.2514	4.2437
	GARCH (2,1)	-6350,93	4.2394	4.2514	4.2437
	GARC11 (2,2)	-6349.78	4.2393	4.2533	4.2443
Asymmetric TARCH Model					
Normal	TARCH (1,1)	-6812.70	4.5466	4.5567	4.5503
	TARCH (1,2)	-6919.73	4.6187	4.6307	4.6230
	TARCH (2,1)	-6788.03	4,5309	4.5429	4.5352
	TARCH (2,2)	-6758.51	4.5118	4.5259	4.5169
Studenl's-t	TARCH (1,1)*	-6281.05	4.1928	4.2048	4.1971
	TARCH (1,2)	-6723.73	4.4887	4.5027	4.4937
	TARCH (2,1)	-6281.17	4.1934	4.2074	4.1984
	TARCH (2,2)	-6889.43	4.5998	4.6158	4.6056
GED	TARCH (1,1)	-6348.27	4.2376	4.2496	4.2419
	TARCH (1,2)	-6348.26	4.2383	4,2523	4.2433
	TARCH (2,1)	-6347.93	4.2380	4.2521	4.2431
	TARCH (2,2)	-6345.37	4.2370	4.2530	4.2428

Results of Parameter Estimation of Volatility Models

To investigate the symmetric features of the monthly and daily crude oil returns, the symmetric ARMA-GARCH models for the monthly returns and the symmetric GARCH models for the daily returns arc are employed, and both results are reported in Table 5. To investigate the asymmetric and leverage effects properties of the monthly and daily crude oil returns, the asymmetric ARMA- TARCH models for monthly returns and asymmetric TARCH models for the daily returns are employed, and both results are reported in Table 6.

Model Diagnostic Checking

To validate the estimated volatility models for both monthly and daily crude oil returns, we employed Engle's LM test. The results are presented in Table 7.

Table 5: Parameter Estimates for Symmetric GARCH Models

Symmetric ARMA (1,1)-GARCH (2,1) Model for Monthly Crude Oil Returns				
Mean Equation				
Variable	Coefficient	Std. Error	z-Statistic	P-value
μ	1.439279	0.720866	1.996596	0.0459
AR (1)	0.197477	0.069755	2.831009	0.0028
MA(1)	0.627840	0.189352	3.315729	0.0000
Variance Equation				
ω	2.502873	0.412145	6.072797	0.0000
a_1	0.095177	0.020304	4.687599	0.0000
a_2	0.224397	0.110640	2.028181	0.0425
β_1	0.596368	0.186026	3.205837	0.0013
v	1.412880	0.188055	7,513136	0.0000
$a_1 + a_2 + \beta_1$	0.915942			
Symmetric GARCH (1,1) Model for Daily Crude Oil Returns (USD)				
Mean Equation				
μ	0.065681	0.028469	2.307072	0,0211
Variance Equation				
ω	0.248337	0.040713	6.099675	0.0000
a_1	0.178130	0.021423	8.315063	0.0000
β_1	0.793226	0.017648	44.9473 7	0.0000
v	4.254656	0.250423	16.98989	0.0000
$a_1 + \beta_1$	0.971356			

Table 6: Parameter Estimates for Asymmetric TARCH Models

Asymmetric ARMA (1,1)-TARCH (2,1) Model for Monthly Crude Oil Returns				
Mean Equation				
Variable	Coefficient	Std. Error	z-Statistic	P-value
μ	0.638457	0.773785	0.825109	0.4093
AR(1)	-0.613279	0.294270	-2.084069	0.0420
MA(1)	0.245402	0.069165	3.548066	0.0000
Variance Equation				
ω	4.698682	1.112250	4,224484	0.0009
a_1	0.232651	0.065452	3.554529	0.0000
γ	0.941848	0.352761	2.669933	0.0076
a_2	0.186773	0.085002	2.197292	0.0280
β_1	0.482110	0.057437	8.393718	0.0000
v.	*8.379826*	*1.828056*	*4.583508*	0.0000
$a_1 + a_2 + \beta_1$	0.901534			
Asymmetric TARCH (1,1) Model for Daily Crude Oil Returns				
Mean Equation				
μ	0.053305	0.028699	1.857389	0.0633
Variance Equation				
ω	0.246576	0.03 8608	6.3 86622	0.0000
a_1	0.115543	0.023311	4.956665	0.0000
γ	0.111970	0.033128	3.379898	0.0007
β_1	0.796341	0.016915	47.07919	0.0000
v	4.307197	0.250691	17.18129	0.0000
$a_1 + \beta_{1I}$	0.911884			

Table 7: Heteroskedasticity Test of ARCH Effects in Estimated Models

Model	F-statistic	P-value	nR2	P-value
Monthly Crude Oil Returns				
ARMA (1,1)-GARCH (2,1)	0.395857	0.5300	0.399091	0.5276
ARMA (1,1)-TARCH (2,1)	0.050530	0.8224	0.051033	0.8213
Daily Crude Oil Returns				
GARCH (1,1)	0.026781	0.8700	0.026799	0.8700
TARCH(1,1)	0.005584	0.9404	0.005588	0.9404

Volatility Mean Reversion and Half-Life

Two tests were employed to test for no mean reversion in volatility. The first test is the DF GLS (ERS) unit root test (see 'fable 2. The results of the unit root test show that the monthly and daily crude oil return series under review are stationary' (there are no unit roots in the data). Any stationary series is also mean reverting, which means that the volatilities of the series will finally revert to their long-run averages. Second, mean reversion in monthly and daily crude oil returns is tested using symmetric ARMA-GARCH models for monthly returns and symmetric GARCH models for daily crude oil returns. In stationary ARMA (1,1)-GARCH (1,1)and GARCH (1,1) models, volatility mean reversion rate is given by the sum («! 4- ft), which is generally close to unity for most financial data. The half-life of a volatility shock measures the average number of lime periods the volatility takes to revert to its long-run average. The absolute value of ($a_1 + \beta_1$) controls the mean reversion speed. Estimates of mean reversion rates and volatility half-lives for monthly and daily crude oil returns are presented in Table 8.

Table 8: Volatility Half-1Jves Results for Symmetric GARCH Models

	Log(2)	$a_1 + \beta_1$	$\log(a_1 + \beta_1)$	$\frac{\log(2)}{\text{Log}(a_1 + \beta_1)}$	$1 - \frac{\log(2)}{\log(a_1 + \beta_1)}$
Monthly Crude Oil Returns					
ARMA-GARCH	0.30103	0.915942	-0.03813	-7,89441	8.894414
ARMA-TARCH	0.30103	0.901534	-0.04502	-6.6869	7.686897
Daily Crude Oil Returns					
GARCH (1,1)	0.30103	0.971358	-0.01262	-23.8521	24.85212
TARCH(1,1)	0.30103	0.91 1881	-0.04006	-7.51413	8.514134

Discussion of Findings

This Section presents discussion of results of the data analysis in this study. Specifically, this section focuses on the discussion of the results of summary statistics and normality measures of the monthly and daily crude oil data in Nigeria, unit root and heteroskedasticity test results, model order and error distribution selection results, parameter estimates of both symmetric and asymmetric GARCH models, volatility mean reversion, and model diagnosis.

Discussion of Results of Summary Statistics and Graphical Examination

The summary statistics and normality measures reported in Table 1 show that the monthly and daily means of crude oil prices in Nigeria are 78.1723 and 78.6217 US Dollars per barrel,, respectively. The monthly and daily means of crude oil returns in Nigeria are 0.1894 percent and 0.0072 percent, respectively. Positive means indicate gains in crude oil prices and returns during the period under review. The standard deviations of both monthly and daily crude oil prices are 26.5285 and 27.9023 US Dollars per barrel, respectively, while the standard deviations of both monthly and daily crude oil returns are 12.9212 percent and 3.5176 percent, respectively, as compared to their monthly and daily mean values. This indicates high levels of dispersions from the monthly and daily averages of crude oil prices and returns in the oil market during the period under review. The higher the standard deviation, the higher the volatility of the market and the riskier the crude is. The wide gaps between the maximum and minimum crude oil prices and returns (the range) give supportive evidence to the high level of variability of oil price changes in the Nigerian oil market.

The monthly and daily crude oil prices exhibit positive skewness. Positive skewness shows that the upper tail of the distribution is thicker than the lower tail, indicating that crude oil prices rise more often than falls. However, monthly and daily crude oil returns exhibit negative skewness. Negative skewness shows that the lower tail of the distribution is thinner than the upper tail, indicating that crude oil returns fall more often than rises. All returns on crude oil exhibit excess kurtosis. AH, the return series have non-normal distributions with high kurtosis values. The Jarque-Bera test statistics rejected the null hypothesis of normality in both monthly and daily crude oil returns with highly significant p-values. This means that the Nigerian crude oil returns during the period of investigation are non-Gaussian and do not follow normal distributions.

From the time plots of the monthly and daily prices of crude oil in Nigeria reported in Figure 1 (left), it is clearly seen that the trend movement in the plots is not smooth. This suggests that the means and variances are heteroskedastic and that the series is non-stationary. We therefore transform the series to natural log returns. - Having transformed the monthly and daily prices of crude oil to monthly and daily log returns, GARCH we now consider the graphical properties of the returns presented in Figure 1 (right).

The time-series plots of the monthly arid daily log returns indicate a smooth trend, showing that the returns are covariance stationary with some period's being riskier than others. The amplitudes of the returns vary over time as large changes in returns tend to be followed by large changes, whereas small changes are followed by small changes. This indicates that returns are driven by the same market forces. Periods of high-volatility clustering imply frequent changes in crude oil prices in the Nigerian economy, whereas periods of low-volatility clustering entail either persistence of constant oil price stock over time or persistence of oil price shocks in Nigeria. Thus, both volatility clustering and shock persistence are evidenced in log returns for crude oil prices.

Discussion of Results of Unit Root and Heteroskedasdiy Tests

The DF GLS unit root test results reported in Table 2 indicate that the monthly and daily crude oil prices for the period of investigation are all non-stationary in levels (contains unit root in level). This is demonstrated by the DF GLS test statistics being higher than the corresponding critical values at the 1% and 5% levels of significance.

However, the DF GLS unit root test results for the monthly and daily crude oil returns scries reported in Table 2 show that both the monthly and daily crude oil returns are stationary. This is demonstrated by the DF GLS test statistics being less than the corresponding critical values at the 1% and 5% levels of significance. Hence, it can be concluded that the monthly and daily crude oil prices in Nigeria are non-stationary, whereas their corresponding monthly and daily returns are stationary.

Engle's LM test of heteroskedasticity for the ARCH effect reported in the lower panel of Table 2 strongly rejected the null hypothesis of no ARCH effect in the residuals of crude oil returns. The p-values of the F-statistics and nR2 are all highly statistically significant at the 1% marginal significance level. This means that the monthly and daily crude oil returns in Nigeria under review exhibit heteroskedasticity (time-varying conditional variance) and can only be modeled using the ARCH or GARCH family models.

Discussion of the Model Selection Results

From the model order and error distribution selection results reported in Tables 3 and 4, for both monthly and daily crude oil return series, three different error distributions are considered; these are the normal distribution (ND), the student-t distribution (STD), and the generalized error distribution (GED). For monthly crude oil returns, the symmetric ARMA (1,1)-TARCH (2,1)model with GED and the asymmetric ARMA (1,1)-GARCH (2J) model with student's-t distribution are selected for modeling the monthly crude oil returns under consideration of minimum information criteria and maximum log likelihoods.

For the daily crude oil returns, the symmetric GARCH (1,1) model with Student's t distribution (STD) and the asymmetric TARCH (1.1) model with Student's t distributions were suitable for modeling the daily crude oil returns series in Nigeria based on minimum information criteria and maximum log likelihoods.

Based on the types of error distributions selected for modding volatility of both monthly and daily crude oil returns in Nigeria, it suffices to say that the Nigerian crude oil return series are fat-tailed, as only heavy-tailed error distributions were suitably selected for modeling them.

Discussion of Results of Models Estimation and Models Diagnosis

The results of volatility estimates presented in Table 5 depict the coefficients of both the mean and conditional variance equations of the symmetric ARMA (1,1)-GARCH (2,1) and GARCH (1,1) models for the monthly and daily crude oil returns, respectively.

The result of the mean equation (4.1) shows that the intercept (g) is positively related to monthly crude oil log returns and statistically significant at the 5% level of significance. This implies that the predicted value of the monthly crude oil log return will be approximately 1.4% if all other explanatory variables remain constant. The AR and MA slope coefficients of the model are all statistically significant at 5% significance-Bevels. The estimated model also satisfied the stationarity condition because the sum of AR and MA terms was less than unity. That is $ø_2 + ø_1 = 0.197477 + 0.627840 = 0.825317 < 1$. This indicates that the estimated ARMA (1,1) model is stationary.

In the conditional variance equations, all estimated parameters are highly statistically significant at 1% marginal significance levels and satisfy the models' non-negativity restrictions. The significance of ARCH parameters (a_1) indicates that news on volatilities from previous periods has explanatory power for current volatilities. In the same way, the statistical significance of the GARCH parameters (β_1) does not only indicate that news -about volatilities from previous periods have explanatory powers on current volatilities but also suggests volatility clustering in the monthly and daily returns of the crude oil series.

For the conditional variance equations of both the monthly symmetric ARMA (1,1)-GARCH (2,1) model and the daily symmetric GARCH (1,1) model, volatility persistence is measured by the sum ($a_1 + \beta_1$). From the estimates in Table 5, volatility persistence for both the monthly and daily crude oil returns are ($a_1 + a_2 + \beta_1 < 1$) for the ARMA (1,1)-GARCH (2,1) and ($a_1 + \beta_1 < 1$) for GARCH (1,1) models respectively. Because the sums of the volatility persistence coefficients are all less than unity, the conditional variances are all mean reverting and stable, and the entire variance processes are stationary and predictable. However, volatility persistence coefficients are quite high because the sums of ARCH and GARCH coefficients are very close to 1. High volatility persistence implies that average variance will remain high because increases in conditional variance due to effects of volatility shocks decay only slowly.

From the results of the asymmetric ARMA (1,1)-TARCH (1,1) model for monthly crude oil returns and TARCH (1,1) model for daily crude oil returns presented in Table 6, all parameters of the estimated models in the variance equations are statistically significant at 5% levels of significance. The significance of the ARCFI and GARCH terms indicates that previous square-error terms significantly affect' volatility and that past volatility of crude oil returns also influences current volatilities. The ARMA (RIRTARCH (1.1) and TARCH (1,1) models are also stationary because the sums of the ARCH and GARCH terms are less- than unity in all the models. This shows that the conditional variances and volatility shocks are quite persistence and that the conditional variances are stable, and the crude oil log returns are predictable in the oil market.

The asymmetric and leverage effect parameters for the ARMA (1,1)-TARCH (2,1) model for the monthly returns and for the TARCH (1,1) model for the daily crude oil returns are all positive and statistically significant for both the monthly and daily crude oil returns. The positive and significant values of the leverage effect parameter (y) in the asymmetrical models suggest evidence for the existence of asymmetry and leverage effects. This result indicates that bad news (negative shocks) increases volatility more than good news (positive shocks) for the same magnitude. Thus, this study found empirical evidence for the existence of an asymmetry and leverage effect in both monthly and daily crude oil returns in Nigeria.

In estimating GARCH family models with heavy-tailed distributions, such as the student's t distribution, the shape parameter, (v) must be greater than 2 for tire distributions to be fat-tailed. In addition, when estimating GARCH models with the generalized error distribution (GED), the shape parameter, (v) must be less than 2 for the distributions to be fat-tailed. From the results of GARCH family model estimation presented in Tables 5 and 6, the shape parameter ($v > 2$) for all the GARCH models estimated with STD are ($v < 2$) for all the GARCH models estimated using GED, indicating that the crude oil returns under the period of investigation are fat-tailed (leptokurtic).

From the heteroskedasticity test result for ARCH effects reported in Table 7 for the estimated GARCH (1,1), ARMA (1,1)-GARCH (2,1), TARCH (1,1), and ARMA (1,1)-TARCH (2,1)models for the monthly and daily erode, oil returns, it is clearly shown that the GARCH family models have captured all the ARCH effects in the residuals of the crude oil return series. This is indicated by the p-values of -the ARCH LM test statistics, which are highly statistically insignificant. This shows that the estimated GARCH-type models' arc is good, adequate, valid, and accurate in describing the volatility of crude oil returns in Nigeria.

Discussion of Result of Volatility Mean Reversion and Half-Life

The volatility half-life measures the average number of months and days the volatility shock takes to decrease by half to its original value.

From the results of volatility half-lives reported in Table 8, the monthly crude oil returns have volatility half-lives of approximately 9 months and 8 months as modeled by the symmetric ARMA (1,1)-GARCH (2,1) and asymmetric ARMA (1,1)-TARCH (2,1) models, respectively. The daily crude oil returns have volatility half-lives of approximately 25 days and 9 days as modeled by the symmetric GARCH (1, 1) and asymmetric TARCH (1, 1) models, respectively. Both the monthly and daily crude oil returns modeled by different volatility models revert to long-run average values. Mean reverting of oil prices and stocks represents a good opportunity for long-term investment by both local and foreign investors.

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