

## **AN EMPIRICAL INVESTIGATION OF THE CHAOS BASED BANKRUPTCY MODEL FOR PREDICTING COVID-19 RELATED BANKRUPTCIES**

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### **Article Info**

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### **Abstract**

The COVID-19 pandemic has caused unprecedented damage to businesses across various industries, leading to a surge in bankruptcy filings worldwide. To predict COVID-19-related bankruptcies, this study examines the effectiveness of the Chaos Based Bankruptcy Model in forecasting bankruptcy using daily stock market returns. The Chaos Based Bankruptcy Model measures the degree of chaos in a system using the Lyapunov exponent calculated from the time series of daily stock prices. The model is based on the hypothesis that unhealthy systems exhibit less chaos than healthy systems.

Using data from before the pandemic, the binary logistic regression model successfully predicted the bankruptcy status of a company 70.8% of the time. The model compares the Lyapunov exponents of firms approaching bankruptcy with those of non-bankrupt pair match firms, based on the newer NAICS code, to identify firms that are more likely to declare bankruptcy. The coefficient of the model's Lyapunov exponent variable was -8.918, significant at 0.024 levels, indicating that the fewer the levels of chaos observed in the stock price trends of a firm, the higher the probability of bankruptcy.

This paper provides a novel method of bankruptcy prediction, which has not been explored in previous studies. Furthermore, this method has the potential to identify high-risk businesses in advance of financial stress, allowing companies to take preventive measures and avoid filing for bankruptcy. Future research will focus on applying this model to specific industries to predict bankruptcy risk in those industries, which will enable policymakers and investors to make informed decisions.

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## INTRODUCTION

In 1996, two of this study's authors published "A Chaos Approach to Bankruptcy Prediction" (Lindsay and Campbell). That paper used a statistic to quantify chaos, the Lyapunov exponent, estimated from daily stock market return time series data from bankrupt firms and pair match firms based on their SIC (Standard Industry Classification) codes. That study showed that firms approaching bankruptcy exhibited less chaos than pair match firms not approaching bankruptcy. This distinction was used to construct a single variable bankruptcy prediction model with Type 1 and Type 2 error rates of 35 percent. Since publication, the article has been cited sixty-six times.

In 2019, the current study's authors published "The Chaos Based Bankruptcy Model – Current Status" in the *Journal of Accounting and Finance*. That study utilized a binary logistic regression model and used data from 2009 through 2014. It obtained comparable results to the 1996 study.

The COVID-19 pandemic is an exogenous event that the stock market had no way of anticipating and that impacted firms across the economy simultaneously. The current study attempts to use the Chaos Based Bankruptcy Model to predict COVID-19 related bankruptcies, using data from before the pandemic.

## LITERATURE REVIEW

On December 31, 2019, the World Health Organization (WHO) was informed of pneumonia-like cases of unknown cause in Wuhan City, China. A novel coronavirus was identified as the cause by Chinese authorities on January 7, 2020 and was temporarily named "2019-nCoV." [<https://www.euro.who.int/en/health-topics/health-emergencies/coronavirus-covid-19/novel-coronavirus-2019-ncov>]

This is a brief history of COVID-19 in 2020: On January 20, the Centers for Disease Control and Prevention of the U.S. Department of Health and Human Services (CDC) disclosed the first U.S. laboratory confirmed case of COVID-19 from samples taken on January 18 in Washington state. On March 13, President Trump declared a nationwide emergency. On March 15, U.S. states began to pause public participation activities to prevent the spread of the disease. On March 28, the White House extended social distancing measures until the end of April. On April 3, the CDC announced mask wearing guidelines and recommended that everyone wear a mask when outside of the home. By April 13, most U.S. states reported widespread cases of COVID-19.

The impact on the economy was swift and devastating. By May 9, the U.S. unemployment rate reached 14.7 percent, the worst rate since the Great Depression. Over twenty million people were no longer working. The hospitality, leisure, and healthcare industries took the greatest hits. On June 8, the World Bank stated that COVID-19 would plunge the Global Economy into the worst recession since World War II. [<https://www.cdc.gov/museum/timeline/covid19.html>]

There is an extensive bankruptcy prediction literature most of which comprises tests using various prediction models. Bellovary, Giacomino and Akers (2007) examine 165 models for assessing bankruptcy. Regardless of the methodology used, two issues recur in the bankruptcy prediction literature: misclassification errors and tests for external validity.

Of the misclassification errors observed using the various models, there are two types. A Type 1 error misclassifies a firm which actually will go bankrupt as one which will not go bankrupt. A Type 2 error misclassifies a firm which will not go bankrupt as one which will, indeed, become bankrupt. Type 1 errors have been estimated to be 35 times costlier to decision makers than Type 2 errors (Altman, 1977).

Jones (1987) discusses the need to use a validation method to test any newly developed model. Once a model has been developed using one set of data, it should be tested using an independent set of data. Often this is accomplished by testing the model on a hold-out sample. However, in studies with a small sample size, bootstrapping is often used as an alternative. Bootstrapping is a testing technique that estimates the properties of the sampling distribution from the sampling data. It does this through random sampling with replacement of the sampling data (Field, 2013).

Bellovary et al. (2007) show that most prediction models are based on a cross-sectional analysis which compares different firms on the basis of financial variables reported at a specific point in time. Zmijewski (1983) identified the 75 individual ratios most often used in distress prediction studies. No theory has yet been successfully defended to suggest why some variables would be preferable to others (Foster, 1986).

Only occasionally has a bankruptcy prediction study combined a market-based variable with ratios derived from financial statements (White et al, 1994). Further, there is no theoretical reason why a time series approach could not be used. Prior to Lindsay and Campbell (1996), no bankruptcy prediction study used a time series methodology based upon chaos, which is also known as non-linear dynamics.

Chaotic systems (Yorke, 1976) appear to be random, when in actuality they are deterministic and predictable over short periods of time. They are extremely sensitive to initial conditions, a phenomenon known as the Butterfly Effect. Chaotic systems have proven quite successful in the prediction of certain endogenously determined catastrophic system failures. Goldberger (1990) applied the concept to predict myocardial infarction. Stock returns have also been shown to exhibit chaotic behavior (Peters, 1991).

Etheridge and Sriram (1993) argue persuasively that economics and finance researchers have already successfully used chaos theory to study stock market behavior.

### **HYPOTHESIS DEVELOPMENT**

This study uses Lyapunov exponents to measure chaos. The exponent measures the rapidity with which a system becomes unpredictable. The larger the exponent, the sooner the system becomes unpredictable. Any system with a positive Lyapunov exponent is chaotic. Goldberger (1990) suggests that healthy systems exhibit more chaos than unhealthy systems. The hypothesis of the study is:

*H<sub>1</sub>: The Lyapunov exponent estimated from the stock market returns of pre-pandemic firms approaching bankruptcy will be lower than the exponents of pre-pandemic firms not approaching bankruptcy.*

### **METHODOLOGY**

This study predicts bankruptcies that occurred during the pandemic by using stock market return data from a period of time just prior to the discovery of the existence of a new coronavirus, the COVID-19 virus, infecting and passing between humans. The World Health Organization was informed of an epidemic of unknown cause in Wuhan, China on December 31, 2019. Therefore, the stock market return data used in this study were from the four-year time period ending November 29, 2019. One thousand daily returns were collected to provide a sufficient number of data points to calculate chaos statistics.

On February 25, 2020, the CDC announced that COVID-19 was headed toward pandemic status [<https://www.ajmc.com/view/a-timeline-of-covid19-developments-in-2020>]. Once this information was public, it would have been discounted into stock prices. Firms which filed for Chapter 11 bankruptcy protection between March 1 and December 31, 2020, were identified from *Kiplinger*, *Investor's Business Daily* and *Retail Dive.com*. The bankrupt firms were cross-referenced with daily stock returns listed in the Thomson Reuters Eikon database. Bankrupt firms which lacked a complete set of Eikon daily return data were removed from the sample. To create a control sample, each firm in the bankrupt sample was matched by four-digit NAICS (North American Industry Classification System) code with a non-bankrupt firm to create a pair match. A comparison of the older SIC system to the new NAICS system can be seen at <https://siccode.com/page/history-of-sic-codes>.

The Lyapunov exponent for each bankrupt firm and its pair match firm were calculated using the Chaos Data Analyzer software package (Sprott and Rowlands, 1992). This study's hypothesis leads to the expectation that bankrupt firms will exhibit less chaos, and hence, will have lower Lyapunov exponents than their corresponding pair match firms. This study uses a binary logistic regression model where the Lyapunov exponent is the independent variable (covariate) and the categorical variable of not bankrupt/bankrupt (coded as 0/1) is the dependent variable.

### **RESULTS**

The test sample is comprised of 24 firms that declared Chapter 11 bankruptcy between March 1 and December 31, 2020, and their NAICS code pair match firms. Daily stock market returns for each of the 48 firms were

obtained from Eikon for the four-year period ending November 29, 2019. This period includes 1,000 daily returns. The impact of COVID-19 could not have been discounted into this sample of stock returns, since the World Health Organization was not aware of an epidemic in Wuhan, China until December 31, 2019. The returns were used to calculate Lyapunov exponents for both the test firms and the pair match firms. Table 1 presents the 24 bankrupt companies together with their chapter 11 filing dates, NAICS codes, pair matches, and Lyapunov exponents.

Table 2 shows descriptive statistics of the independent and dependent variables. Table 3 presents the Pearson correlation coefficients between the two variables. The correlation between the dependent variable, not bankrupt/bankrupt, and the independent variable, Lyapunov exponent, is -0.360, and it is significant at the 0.05 level. The negative correlation supports the hypothesis that the Lyapunov exponent estimated from the stock market returns of pre-pandemic firms approaching bankruptcy will be lower than the exponents of pre-pandemic firms not approaching bankruptcy.

Table 4 presents the t-test of the differences between the Lyapunov exponents of bankrupt and pair match firms. The mean of the difference is negative and is significant at the .008 level. These results support the hypothesis.

It is inappropriate to use a linear regression when the dependent variable in a model is a 0/1 categorical variable since the function is discontinuous. The correct methodology is to use a binary logistic regression. In linear regressions, R square is the appropriate measure of how well the model fits the data. In binary logistic regressions, a pseudo-R square serves this function (Field, 2013). Table 5 displays the model's Cox & Snell R Square of 0.144 and the Nagelkerke R square of 0.192.

Table 6 displays the binary logistic regression output. In the model, the Lyapunov exponent is the sole covariant. The coefficient on the log of the Lyapunov exponent variable (B) is -8.918, which is significant at the 0.024 level. These results support the hypothesis.

The binary logistic classification table for the model is presented in Table 7. The model correctly predicts the bankruptcy status of a company 70.8 percent of the time. A naive model, such as a coin toss, would obtain a 50 percent success rate. The model successfully predicts which specific firms will go bankrupt 54.2 percent of the time, and it successfully predicts firms that will not go bankrupt 87.5 percent of the time.

The sample is 24 bankrupt firms and their pair matches. Due to the limited sample size, a set-aside sample was not created. Instead, bootstrapping was used to generate 1,000 samples. The results of bootstrapping for the model are shown in Table 8. Bootstrapping does not change the values of the estimated coefficients of the variables; it only impacts the coefficients' significance and their confidence intervals. The estimated coefficient on the Lyapunov exponent variable remained significant at 0.05 level.

## SUMMARY AND CONCLUSIONS

The outcomes of this study are consistent with the notion that unhealthy systems display less chaos than healthy systems. The results of the binary logistic regression model support the hypothesis that firms approaching bankruptcy display less chaos, as measured by the Lyapunov exponent, than pair match firms not approaching bankruptcy. The Nagelkerke R square of the model is 0.192. The coefficient of the model's Lyapunov exponent variable was significant at the 0.024 level, and the bankruptcy status of a company was correctly predicted 70.8 percent of the time.

Future research will focus on applying the model to specific industries.

**TABLE 1 LYAPUNOV EXPONENTS OF BANKRUPT AND PAIR-MATCH FIRMS**

<b>Bankrupt Firm Name</b>	<b>Filing Date</b>	<b>NAICS Code</b>	<b>Bankrupt Firm Lyapunov</b>	<b>Pair Match Name</b>	<b>Pair Match Firm Lyapunov</b>
<b>Real Goods Solar</b>	3/5/2020	423720	0.313	<b>Watsco, Inc.</b>	0.596

<b>Bluestem Brands</b>	3/9/2020	454110	0.555	<b>Wayfare</b>	0.582
<b>Foresight Energy Partnership</b>	3/10/2020	212111	0.419	<b>Alliance Resource Partners</b>	0.558
<b>Omagine</b>	3/10/2020	237210	0.494	<b>LGI Homes, Inc.</b>	0.628

<b>Bankrupt Firm Name</b>	<b>Filing Date</b>	<b>NAICS Code</b>	<b>Bankrupt Firm Lyapunov</b>	<b>Pair Match Name</b>	<b>Pair Match Firm Lyapunov</b>
<b>Generation Zero Group</b>	3/13/2020	518210	0.424	<b>Seagate Tech.</b>	0.57
<b>Globe Photos</b>	3/14/2020	541921	0.588	<b>Gartner, Inc.</b>	0.595
<b>BioRestorative Therapies</b>	3/20/2020	541715	0.460	<b>Iqvia Holdings Inc.</b>	0.606
<b>Carbo Ceramics</b>	3/29/2020	561990	0.599	<b>Quinstreet Inc.</b>	0.567
<b>Broadvision</b>	3/30/2020	443142	0.574	<b>Best Buy Inc.</b>	0.553
<b>Whiting Petroleum</b>	4/1/2020	424720	0.627	<b>Sprague Resources LP</b>	0.547
<b>Yuma Energy</b>	4/15/2020	454310	0.282	<b>Ferrellgas LP</b>	0.272
<b>Eco-Stim Energy Solutions</b>	4/16/2020	213112	0.462	<b>Halliburton Co</b>	0.645
<b>United Cannabis</b>	4/20/2020	561499	0.563	<b>Document Technologies</b>	0.447
<b>Diamond Offshore Drilling</b>	4/26/2020	213111	0.588	<b>Patterson-UTI Energy Inc.</b>	0.649
<b>Stage Stores</b>	5/11/2020	448140	0.549	<b>Nordstrom Inc.</b>	0.588
<b>Intelsat</b>	5/13/2020	517919	0.607	<b>Comcast</b>	0.623
<b>J. C. Penney</b>	5/15/2020	452210	0.608	<b>Target</b>	0.595
<b>Centric Brands</b>	5/18/2020	315240	0.056	<b>Cintas Corp</b>	0.608
<b>GNC</b>	6/23/2020	446191	0.473	<b>Natural Grocers</b>	0.453
<b>RTW Retailwinds</b>	7/13/2020	448190	0.512	<b>Abercrombie &amp; Fitch</b>	0.579
<b>Ascena</b>	7/23/2020	448120	0.621	<b>Express Inc.</b>	0.609
<b>Tailored Brands</b>	8/2/2020	448110	0.504	<b>American Eagle Outfitters</b>	0.607
<b>Stein Mart</b>	8/12/2020	452210	0.422	<b>Burlington Stores Inc.</b>	0.572
<b>Francesca's</b>	12/3/2020	448120	0.415	<b>Express Inc.</b>	0.654

TABLE 2 DESCRIPTIVE STATISTICS

<u>Variable</u>	<u>N</u>	<u>Minimum</u>	<u>Maximum</u>	<u>Mean</u>	<u>Standard Deviation</u>
Not Bankrupt/Bankrupt	48	0	1	0.50	0.505

Lyapunov Exponent	48	0.056	0.654	0.523	0.116
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**TABLE 3 PEARSON CORRELATION COEFFICIENTS**

N = 48

	<u>Not Bankrupt/ Bankrupt</u>	<u>Lyapunov Exponent</u>	
Not Bankrupt /Bankrupt	1	0.360*	
Lyapunov Exponent.		1	

\* Correlation is significant at the 0.05 level.

**TABLE 4****T-TEST OF THE DIFFERENCES BETWEEN THE LYAPUNOV EXPONENTS OF BANKRUPT AND PAIR MATCH FIRMS**

N = 48

<u>Mean</u>	<u>Standard Deviation</u>	<u>t</u>	<u>Two-Sided p</u>
-0.08	0.14091	-2.88	0.008

**TABLE 5****PSEUDO R SQUARE OF THE BINARY LOGISTIC REG****RESSION**

<u>Cox &amp; Snell R Square</u>	<u>Nagelkerke R Square</u>
0.144	0.192

**TABLE 6****BINARY LOGISTIC REGRESSION OUTPUT**

N=48

	<u>B</u>	<u>S.E.</u>	<u>Wald</u>	<u>df</u>	<u>Sig.</u>	<u>Exp(B)</u>
Lyapunov	-8.918	3.957	5.078	1	.024	000
Constant	4.810	2.201	4.775	1	.029	122.770

**TABLE 7****BINARY LOGISTIC CLASSIFICATION TABLE**

N = 48

	<u>Observed</u>	<u>Predicted</u>	
	Not Bankrupt	Bankrupt	Percent Correct
Not Bankrupt	21	3	87.5
Bankrupt	11	13	54.2
Overall Percentage			70.8



**TABLE 8 BOOTSTRAP TEST TO VALIDATE MODEL**

		<u>95 Percent Confidence Interval</u>		
	<u>B</u>	<u>Sig.</u>	<u>Lower</u>	<u>Upper</u>
Lyapunov Exponent	-8.918	0.046	-26.666	-2.241
Constant	4.810	0.060	1.099	14.583
Results are based on 1,000 bootstrap samples.				

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