International Journal of Allied Sciences (IJAS)
Volume.13, Number 1; January-2022;
ISSN: 2836-3760 | Impact Factor: 6.15
https://zapjournals.com/Journals/index.php/Allied-Sciences
Published By: Zendo Academic Publishing

# THE $\chi^2$ OF TENSOR PRODUCTS İN DOUBLE ORLİCZ SEQUENCE SPACES

**[1]Ayhan Esi**

**Article Info**

**Keywords:** Analytic sequence, double sequences, $\chi^2$ space, Musielak-Orlicz function, p-metric space, Banach metric lattice, positive tensor product.

**Abstract**

Let $X$ be a Banach lattice and $\chi_M{}^2$ be an double gai Orlicz sequence space associated to an Orlicz function with the $\Delta_2 -$ condition. In this paper discuss the some general topological properties of tensor products.

2000 *Mathematics Subject Classification*. 46B42, 46B28.

## Introduction

Throughout $w$, $\chi$, $\Gamma$ and $\Lambda$ denote the classes of all, gai,entire and analytic scalar valued single sequences, respectively.

We write $w^2$ for the set of all complex sequences $(x_{mn})$, where $m, n \in \mathbb{N}$, the set of positive integers. Let $(x_{mn})$ be a double sequence of real or complex numbers. Then the series

$\sum^{\infty}_{m,=1} x_{mn}$is called a double series. The double series $\sum^{\infty}_{m,=1} x_{mn}$is said to be convergent if and only if the double sequence $(S_{mn})$is convergent, where

$S_{mn} = \sum_{m,ni,j=1} x_{ij}(m, n = 1,2,3, \ldots)$ .

We denote $w^2$ as the class of all complex double sequences $(x_{mn})$. A sequence $x = (x_{mn})$ is said to be double analytic if

$sup_{m,}|x_{mn}|_{1/m+n} < \infty$.

The vector space of all prime sense double analytic sequences are usually denoted by $\Lambda^2$. A sequence $x = (x_{mn})$ is called double entire sequence if

$|x_{mn}|^{1/m+n} \to 0$ as $m, n \to \infty$.

The vector space of all prime sense double entire sequences are usually denoted by $\Gamma^2$. The space $\Lambda^2$ and $\Gamma^2$ is a metric space with the metric

$$(x, y) = sup_{m,n}\{|x_{mn} - y_{mn}|^{1/m+n}: m, n: 1,2,3, \ldots\}, \qquad (1)$$

for all $x = \{x_{mn}\}$ and $y = \{y_{mn}\} in \quad \Gamma^2$. Let $\phi = \{finite \quad sequences\}$.

A sequence $x = (x_{mn})$ is called double gai sequence if $((m + n)!\,|x_{mn}|)^{1/m+n} \to 0$ as $m, n \to \infty$. The double gai sequences will be denoted by $\chi^2$.

Let $M$ and $\Phi$ be mutually complementary Orlicz functions. Then, we have

(i) For all $u, y \geq 0$,

$uy \leq (u) + \Phi(y), (Young's \quad inequality)[see, Kampthan\ etal., (1981)]$ (2) (ii) For all $u \geq 0$,

[1] Malatya Turgut Ozal University, Engineering Faculty, Department of Basic Engineering Sciences, 44040, Malatya, Turkey.

$$u\eta(u) = M(u) + \Phi(\eta(u)). \qquad (3)$$

(iii) For all $u \geq 0$, and $0 < \lambda < 1$,

$$M(\lambda u) \leq \lambda M(u). \qquad (4)$$

A sequence $M = (M_{mn})$ of Orlicz function is called a Musielak-Orlicz function. A sequence $g = (g_{mn})$ defined by $g_m(v) = sup\{|v|u - M_{mn}(u) : u \geq 0\}, m, n = 1,2, \cdots$
is called the complementary function of a Musielak-Orlicz function $M$. For a given Musielak
Orlicz function $M$, the Musielak-Orlicz sequence space $t_M$ is defined by
$M_f = \{x \in w^2 : I_M(|x_{mn}|)^{1/m+n} \to 0 \quad as \quad m, n \to \infty\}$,
where $I_M$ is a convex modular defined by
$I_M(x) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} M_{mn}(|x_{mn}|)_{1/m+n}, x = (x_{mn}) \in t_M$.
We consider $t_M$ equipped with the Luxemburg metric

$$(x, y) = sup_{m,n} \left\{ inf \left( \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} M_{mn} \left( \frac{|x_{mn}|1/m+n}{\quad\quad}_{mn} \right) \right) \leq 1 \right\}.$$

The postivity perspective, it is know that the projective tensor and the injective tensor product of two Banach lattices are, in general not Banach lattices.

Throughout $w$, $\chi$ and $\Lambda$ denote the classes of all, gai and analytic scalar valued single sequences, respectively. We write $w^2$ for the set of all complex double sequences $(x_{mn})$, where $m, n \in \mathbb{N}$, the set of positive integers.Then, $w^2$ is a linear space under the coordinate wise addition and scalar multiplication.

**2 Notations**

For a vector space $X$, a vector $\bar{x} = (x_{ij})_j \in X^{\mathbb{N}\times\mathbb{N}}$ and $n \in \mathbb{N}$, we write $\bar{x}(\leq n)$ is a two dimensional matrix from first term to nth term and remaining term zero. and $\bar{x}(> n)$ is a two dimensional matrix from first term to nth term zero and start with (n+1)th term.

If $X$ is an ordered set, the usual order on $X^{\mathbb{N}\times\mathbb{N}}$ is defined by $\bar{x} = (x_{ij})_{ij} \geq 0 \Leftrightarrow x_{ij} \geq 0$ for each $i, j \in \mathbb{N}$. for Banach lattice $X$, $X^*$ denotes its dual space, $B_X$ denotes its closed unit ball, and $X^+$ denotes its positive cone.

**3 Definitions and Preliminaries**

A sequence $x = (x_{mn})$is said to be double analytic if
$sup_{m,n}|x_{mn}|^{\frac{1}{m+n}} < \infty$. The vector space of all double analytic sequences is usually denoted by
$\Lambda^2$. A sequence $x = (x_{mn})$ is called double entire sequence if $|x_{mn}|^{\frac{1}{m+n}} \to 0$ as $m, n \to \infty$. The vector space of double entire sequences is usually denoted by $\Gamma^2$. A sequence $x = (x_{mn})$
is called double gai sequence if $((m+n)!|x_{mn}|)^{\frac{1}{m+n}} \to 0$ as $m, n \to \infty$. The vector space of double gai sequences is usually denoted by $\chi^2$. The space $\chi^2$ is a metric space with the metric

$d(x, y) = sup_{m,n}\{((m+n)!\,|x_{mn} - y_{mn}|)^{\frac{1}{m+n}} : m, n, k : 1,2,3, \ldots\}$ (5) for all $x = \{x_{mn}\}$ and $y = \{y_{mn}\}$ in $\chi^2$.

Let $n \in \mathbb{N}$ and $X$ be a real vector space of dimension $w$, where $n \leq m$. A real valued function $d_p(x_1, \ldots, x_n) = \| (d_1(x_1, 0), \ldots, d_n(x_n, 0)) \|_p$ on $X$ satisfying the following four conditions:

(i) $\| (d_1(x_1, 0), \ldots, d_n(x_n, 0)) \|_p = 0$ if and only if $d_1(x_1, 0), \ldots, d_n(x_n, 0)$ are linearly dependent,

(ii) $\| (d_1(x_1, 0), \ldots, d_n(x_n, 0)) \|_p$ is invariant under permutation,

(iii) $\| (\alpha d_1(x_1, 0), \ldots, d_n(x_n, 0)) \|_p = |\alpha| \, \| (d_1(x_1, 0), \ldots, d_n(x_n, 0)) \|_p, \alpha \in \mathbb{R}$

(iv) $d_p((x_1, y_1), (x_2, y_2) \cdots (x_n, y_n)) = (d_X(x_1, x_2, \cdots x_n)^p + d_Y(y_1, y_2, \cdots y_n)^p)^{1/p} for 1 \leq p < \infty$; (or)

(v) $d((x_1, y_1), (x_2, y_2), \cdots (x_n, y_n)) := \sup\{d_X(x_1, x_2, \cdots x_n), d_Y(y_1, y_2, \cdots y_n)\}$,

for $x_1, x_2, \cdots x_n \in X, y_1, y_2, \cdots y_n \in Y$ is called the $p$ product metric of the Cartesian product of $n$ metric spaces is the $p$ norm of the $n$-vector of the norms of the $n$ subspaces. A trivial example of $p$ product metric of $n$ metric space is the $p$ norm space is $X = \mathbb{R}$ equipped with the following Euclidean metric in the product space is the $p$ norm:

$$\| (d_1(x_1, 0), \ldots, d_n(x_n, 0)) \|_E = sup(|det(d_{mn}(x_{mn}, 0))|) = sup \left| \begin{matrix} d_{11}(x_{11}, 0) & d_{12}(x_{12}, 0) & \ldots & d_{1n}(x_{1n}, 0) \\ d_{21}(x_{21}, 0) & d_{22}(x_{22}, 0) & \ldots & d_{2n}(x_{1n}, 0) \\ \vdots & & & \\ d_{n1}(x_{n1}, 0) & d_{n2}, 0(x_{n2}, 0) & \ldots & d_n(x_{nn}, 0) \end{matrix} \right|$$

where $x_i = (x_{i1}, \cdots x_{in}) \in \mathbb{R}^n$ for each $i = 1, 2, \cdots n$.

If every Cauchy sequence in $X$ converges to some $L \in X$, then $X$ is said to be complete with respect to the $p-$ metric. Any complete $p-$ metric space is said to be $p-$ Banach metric space. **3.1 Definition**

**Positive tensor products:** For Banach lattices $X$ and $Y$, let $X \otimes Y$ denote the algebraic tensor product of $X$ and $Y$. For each

$u = \sum_{m=1}^{r} \sum_{n=1}^{s} x_{mn} \otimes y_{mn} \in X \otimes Y$, define $T_u: X^* \to Y$ by $T(x^*) = \sum_{m=1}^{r} \sum_{n=1}^{s} x^*(x_{mn}) y_{mn}$ for each $x^* \in X^+$.

Then injective cone on $X \otimes Y$ is defined to be $C_i = \{u \in X \otimes Y: T_u(x^*) \in Y^* \quad \forall x^* \in X^{*+}\}$. **3.2 Definition**

Let $X \,\bar{\bar{\otimes}}_i\, Y$ denote the completion of $X \otimes Y$ with respect $(.\,,.)$. Then $X \,\bar{\bar{\otimes}}_i\, Y$ with $C_i$ as its positive cone is a Banach lattice called the positive injective tensor product of $X$ and $Y$. The positive cone on $x \otimes Y$ is defined to be

$C_p = \{\sum_{m=1}^{r} \sum_{n=1}^{s} x_{mn} \otimes y_{mn}: r, s \in \mathbb{N}, x_{mn} \in X_+, y_{mn} \in Y_+\}$. We define the following spaces:

For a Banach metric lattice $X$, let

$$\chi_M^2(X) = \{\bar{x} = (x_{ij})_j \in X^{\mathbb{N} \times \mathbb{N}}: (x^*((i+j)!|x_{ij}|)^{1/i+j})_{ij} \in \chi^2, \forall x^* \in X^{*+}\}.$$

The metric defined to be

$$d(x, y) = sup\{\||(x^*((i+j)!\,|x_{ij} - y_{ij}|)^{1/i+j}) : x^* \in B_{X^{*+}}|\|\}, x = (x_{ij})_{ij} \in \chi_M^2(X).$$

Let $\chi_M^2(X) = \{\bar{x} \in \chi_M^3(X): lim_{i,,n} \||((i+j)!\,|\bar{x}_i(>n)|)^{1/i+j}|\| \to 0 \quad as \quad i, j \to \quad \infty\}$, with the metric $(x, y) = sup\{\||((i+j)!|(\bar{x}_{ij} - \bar{y}_{ij})(>n)|)^{1/i+j}|\| \,\forall (\bar{x}_{ij}) \in \chi_M^2(X^{*+})\}$.

## 4 Some New Orlicz sequence spaces of Tensor product

The main aim of this article is to introduce the following sequence spaces and examine the topological and algebraic properties of the resulting sequence spaces. Let $M = (M_{mn})$ be a sequence Musielak-Orlicz functions, $(X, \|(d(\bar{x}_1, 0), d(\bar{x}_2, 0), \cdots, d(\bar{x}_{n-1}, 0))\|_p)$ be a $p-$ metric space, and consider $\mu_{mn}(\bar{x}) =$

$\||((i+j)!|\bar{x}_{ij}(>n)|)^{1/i+j}\||$ and $\eta_{mn}(\bar{x}) = \|||\bar{x}_{ij}(>n)|^{1/i+j}\||$.

We define the following sequence spaces as follows: $[\chi_{M}{}^{2}, \|(d(\bar{x}_1,0), d(\bar{x}_2,0), \cdots, d(\bar{x}_{n-1},0))\|_p] = \lim_{mn\to\infty} \{\sum_m \sum_n [M_{mn}(\|\mu_{mn}(\bar{x}), (d(\bar{x}_1,0), d(\bar{x}_2,0), \cdots, d(\bar{x}_{n-1},0))\|_p)] = 0\}$, and

$[\Lambda^2{}_M, \|(d(\bar{x}_1,0), d(\bar{x}_2,0), \cdots, d(\bar{x}_{n-1},0))\|_p] = sup\, \{\sum_m \sum_n [M_{mn}(\|\eta_m(\bar{x}), (d(\bar{x}_1,0), d(\bar{x}_2,0), \cdots, d(\bar{x}_{n-1},0))\|_p)] < \infty\}$.

## 5 Main Results

### 5.1 Theorem

Let $M = (M_{mn})$ be a sequence Musielak-Orlicz functions, the tensor product of Orlicz sequence spaces $[\chi_{M}{}^{2}{}_{\mu}, \|(d(x_1), d(x_2), \cdots, d(x_{n-1}))\|_p]$ and $[\Lambda^2{}_{M\eta}, \|(d(x_1), d(x_2), \cdots, d(x_{n-1}))\|_p]$ are linear spaces.

**Proof:** It is routine verification. Therefore the proof is omitted.

### 5.2 Theorem

Let $M = (M_{mn})$ be a sequence Musielak-Orlicz functions, the tensor product of Orlicz sequence space $[\chi_{M}{}^{2}{}_{\mu}, \|(d(x_1), d(x_2), \cdots, d(x_{n-1}))\|_p]$ is a paranormed space with respect to the paranorm defined by

$(x) = \{[M_{mn}(\|\mu_{mn}(x), (d(x_1), d(x_2), \cdots, d(x_{n-1}))\|_p)]\}$.

**Proof:** Clearly $(x) \geq 0$ for $x = (x_{mn}) \in [\chi_{M}{}^{2}{}_{\mu}, \|(d(x_1), d(x_2), \cdots, d(x_{n-1}))\|_p]$ Since $M_m(0) = 0$, we get $g(0) = 0$.

Conversely, suppose that $g(x) = 0$, then

$\{[M_{mn}(\|\mu_{mn}(x), (d(x_1), d(x_2), \cdots, d(x_{n-1}))\|_p)]\}$

Suppose that $\mu_{mn}(x) \neq 0$ for each $m, n \in \mathbb{N} \times \mathbb{N}$. Then $\|\mu_m(x), (d(x_1), d(x_2), \cdots, d(x_{n-1}))\|_p \to \infty$. It follows that

$([M_{mn}(\|\mu_m(x), (d(x_1), d(x_2), \cdots, d(x_{n-1}))\|_p)]) \to \infty$ which is a contradiction. Therefore $\mu_{mn}(x) = 0$. Let

$([M_{mn}(\|\mu_m(x), (d(x_1), d(x_2), \cdots, d(x_{n-1}))\|_p)])$

and

$([M_{mn}(\|\mu_m(y), (d(x_1), d(x_2), \cdots, d(x_{n-1}))\|_p)])$.

Then by using Minkowski's inequality, we have

$([M_{mn}(\|\mu_m(x+y), (d(x_1), d(x_2), \cdots, d(x_{n-1}))\|_p)]) \leq$
$([M_{mn}(\|\mu_m(x), (d(x_1), d(x_2), \cdots, d(x_{n-1}))\|_p)]) +$
$([M_{mn}(\|\mu_m(y), (d(x_1), d(x_2), \cdots, d(x_{n-1}))\|_p)])$.

So we have

$(x+y) = \{[M_{mn}(\|\mu_{mn}(x+y), (d(x_1), d(x_2), \cdots, d(x_{n-1}))\|_p)]\}$
$\leq \{[M_{mn}(\|\mu_m(x), (d(x_1), d(x_2), \cdots, d(x_{n-1}))\|_p)]\}$
$+ \{[M_{mn}(\|\mu_m(y), (d(x_1), d(x_2), \cdots, d(x_{n-1}))\|_p)]\}$

Therefore,

$g(x+y) \leq g(x) + g(y)$.

Finally, to prove that the scalar multiplication is continuous. Let $\lambda$ be any complex number. Therefore paranormed by,

$(\lambda x) = \{[M_{mn}(\|\mu_{mn}(\lambda x), (d(x_1), d(x_2), \cdots, d(x_{n-1}))\|_p)]\}$. Hence it is continuous.

This completes the proof.

**5.3 Theorem**

(i) If the sequences of Musielak-Orlicz functions $(M_{mn})$ and $(N_{mn})$ are satisfies $\Delta_2 -$ condition, then $[\chi_{M}{}^2{}_{\mu}, \|\mu_m(x), (d(x_1), d(x_2), \cdots, d(x_{n-1}))\|_p] =$ $[\chi_N{}^2{}_{\mu}, \|\mu_u(x), (d(x_1), d(x_2), \cdots, d(x_{n-1}))\|_p]$.

(ii) If the sequences of Musielak-Orlicz functions $(M_{mn})$ and $(N_{mn})$ are satisfies $\Delta_2 -$ condition, then $[\chi_N{}^2{}_{\mu}, \|\mu_m(x), (d(x_1), d(x_2), \cdots, d(x_{n-1}))\|_p] =$ $[\chi_M{}^2{}_{\mu}, \|\mu_m(x), (d(x_1), d(x_2), \cdots, d(x_{n-1}))\|_p]$

**Proof:** Let the sequences of Musielak-Orlicz functions $(M_{mn})$ and $(N_{mn})$ are satisfies $\Delta_2 -$ condition, we get

$$[\chi_N{}^2{}_{\mu}, \|\mu_m(x), (d(x_1), d(x_2), \cdots, d(x_{n-1}))\|_p] \subset$$

$[\chi_N{}^2{}_{\mu}, \|\mu_m(x), (d(x_1), d(x_2), \cdots, d(x_{n-1}))\|_p]$ (6) To prove the inclusion $[\chi_M{}^2{}_{\mu}, \|\mu_m(x), (d(x_1), d(x_2), \cdots, d(x_{n-1}))\|_p] \subset [\chi_N{}^2{}_{\mu}, \|\mu_{mn}(x), (d(x_1), d(x_2), \cdots, d(x_{n-1}))\|_p]$, let $a \in [\chi_M{}^2{}_{\mu}, \|\mu_{mn}(x), (d(x_1), d(x_2), \cdots, d(x_{n-1}))\|_p]$. Then for all $\{x_{mn}\}$ with $(x_{mn}) \in$ $[\chi_M{}^2{}_{\mu}, \|\mu_m(x), (d(x_1), d(x_2), \cdots, d(x_{n-1}))\|_p]$ we have

$$\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} |x_{mn} a_{mn}| < \infty. \tag{7}$$

Since the Musielak-Orlicz functions sequence $(M_{mn})$ satisfies $\Delta_2 -$ condition, then $(y_{mn}) \in [\chi_M{}^2{}_{\mu}, \|\mu_m(x), (d(x_1), d(x_2), \cdots, d(x_{n-1}))\|_p]$, we get $\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left|\frac{mn \quad mn}{(m+n)!}\right| < \infty_{y\,a}$. by (7). Thus $(a_{mn})$ $\in [\chi_M{}^2{}_{\mu}, \|\mu_m(x), (d(x_1), d(x_2), \cdots, d(x_{n-1}))\|_p] =$ $[\chi_N{}^2{}_{\mu}, \|\mu_m(x), (d(x_1), d(x_2), \cdots, d(x_{n-1}))\|_p]$ and hence $(a_{mn}) \in [\chi_N{}^2{}_{\mu}, \|\mu_m(x), (d(x_1), d(x_2), \cdots, d(x_{n-1}))\|_p]$. This gives that

$$[\chi_M{}^2{}_{\mu}, \|\mu_m(x), (d(x_1), d(x_2), \cdots, d(x_{n-1}))\|_p] \subset$$

$[\chi_N{}^2{}_{\mu}, \|\mu_m(x), (d(x_1), d(x_2), \cdots, d(x_{n-1}))\|_p]$ (8) we are granted with (7) and (8) $[\chi_M{}^2{}_{\mu}, \|\mu_m(x), (d(x_1), d(x_2), \cdots, d(x_{n-1}))\|_p] =$ $[\chi_N{}^2{}_{\mu}, \|\mu_m(x), (d(x_1), d(x_2), \cdots, d(x_{n-1}))\|_p]$

(ii) Similarly, one can prove that $[\chi_N{}^2{}_{\mu}, \|\mu_m(x), (d(x_1), d(x_2), \cdots, d(x_{n-1}))\|_p] \subset$ $[\chi_M{}^2{}_{\mu}, \|\mu_m(x), (d(x_1), d(x_2), \cdots, d(x_{n-1}))\|_p]$ if the sequence $(N_{mn})$ satisfies $\Delta_2 -$ condition. **5.4 Proposition**

The tensor product of Orlicz sequence space $[\chi_M{}^2{}_{\mu}, \|\mu_m(x), (d(x_1), d(x_2), \cdots, d(x_{n-1}))\|_p]$ is not solid.

**Proof:** The result follows from the following example.

**Example:** Consider

$$x = (x_{mn}) = \begin{pmatrix} 1 & 1 & \dots & 1 \\ 1 & 1 & \dots & 1 \\ . & & & \\ . & & & \\ . & & & \\ 1 & 1 & \dots & 1 \end{pmatrix} \in [\chi^2_{M\mu}, \|\mu_{mn}(x), (d(x_1), d(x_2), \cdots, d(x_{n-1}))\|\,]_p. \text{ Let}$$

$$\alpha_{mn} = \begin{pmatrix} (-1)^{m+n} & (-1)^{m+n} & \dots & (-1)^{m+n} \\ (-1)^{m+n} & (-1)^{m+n} & \dots & (-1)^{m+n} \\ . & & & \\ . & & & \\ . & & & \\ (-1)^{m+n} & (-1)^{m+n} & \dots & (-1)^{m+n} \end{pmatrix}, \text{ for all } m, n \in \mathbb{N} \times \mathbb{N}.$$

Then $\alpha_{mn}x_{mn} \notin [\chi_{M\mu}^{2}, \|\mu_{mn}(x), (d(x_1), d(x_2), \cdots, d(x_{n-1}))\| ]$. Hence $[\chi_{M^2\mu}, \|\mu_{mn}(x), (d(x_1), d(x_2), \cdots, d(x_{n-1}))\| ]_p$ is not solid.

### 5.5 Proposition

The tensor product of Orlicz sequence space $[\chi_{M^2\mu}, \|\mu_{mn}(x), (d(x_1), d(x_2), \cdots, d(x_{n-1}))\| ]_p$ is not monotone.

**Proof:** The proof follows from Proposition 5.4.

### 5.6 Theroem

The tensor product of Orlicz sequence spaces $\left[\chi^2_{\mu}, \|(d(x_1), d(x_2), \cdots, d(x_{n-1}))\| \right]_p$ and $\left[\Lambda^2_{\eta}, \|(d(x_1), d(x_2), \cdots, d(x_{n-1}))\| \right]_p$ are not convergence free.

**Example:** Consider, $[M_{mn}(\|\mu_{mn}(x), (d(x_1), d(x_2), \cdots, d(x_{n-1}))\| )] =_p [M(\|\mu_{mn}(x), (d(x_1), d(x_2), \cdots, d(x_{n-1}))\| )] =_p [(\|\mu_{mn}(x), (d(x_1), d(x_2), \cdots, d(x_{n-1}))\| )]_p$ for all $m, n \in \mathbb{N} \times \mathbb{N}$, for $m, n$ are odd, If $m, n$ even, consider the sequence $(x_{mn}) = (mn)^{-(m+n)}$ for all $m, n \in \mathbb{N} \times \mathbb{N}$ belongs to each of $\left[\chi^2_{\mu}, \|(d(x_1), d(x_2), \cdots, d(x_{n-1}))\| \right]_p$ and $\left[\Lambda^2_{\eta}, \|(d(x_1), d(x_2), \cdots, d(x_{n-1}))\| \right]_p$. Consider the sequence $(y_{mn})$ defined by $(y_{mn})^{1/m+n} = m^2n^2$, for all $m, n \in \mathbb{N} \times \mathbb{N}$. Then $(y_{mn})$ neither belongs to $\left[\chi^2_{\mu}, \|(d(x_1), d(x_2), \cdots, d(x_{n-1}))\| \right]_p$ nor $\left[\Lambda^2_{\eta}, \|(d(x_1), d(x_2), \cdots, d(x_{n-1}))\| \right]_p$. Hence the tensor product Orlicz sequence spaces are not convergence free.

## References


M. Basarir and O.Solancan, On some double sequence spaces, *J. Indian Acad. Math.*, **21(2)** (1999), 193-200.

T.J.I'A.Bromwich, An introduction to the theory of infinite series *Macmillan and Co.Ltd.*, New York, **(**1965).

G.H.Hardy, On the convergence of certain multiple series, *Proc. Camb. Phil. Soc.*, **19** (1917), 86-95.

M.A.Krasnoselskii and Y.B. Rutickii, Convex functions and Orlicz spaces, *Gorningen, Netherlands*, **1961**.

J.Lindenstrauss and L.Tzafriri, On Orlicz sequence spaces, *Israel J. Math.*, **10** (1971), 379-390.

H. Nakano, Concave modulars, *J. Math. Soc. Japan*, **5**(1953), 29-49.

P.K. Kamthan and M. Gupta, Sequence spaces and series, Lecture notes, Pure and Applied Mathematics, *65 Marcel Dekker, In c., New York* , **1**981.

N.Subramanian and U.K.Misra, Characterization of gai sequences via double Orlicz space, *Southeast Asian Bulletin of Mathematics*, **35(4)** (2011), 687-697.

N.Subramanian, B.C.Tripathy and C.Murugesan, The double sequence space of $\Gamma^2$, *Fasciculi Math.*, **40**, (2008), 91-103.

N.Subramanian, B.C.Tripathy and C.Murugesan, The Ces$a'$ro of double entire sequences, *International Mathematical Forum*, **4(2)**(2009), 49-59.

Qingying Bu, Donghai Ji and Yongjin Li, Copies of $\ell_1$ in positive tensor products of Orlicz sequence spaces, *Quaestiones Mathematicae*, **34**, (2011), 407-415.