Global Journal of Chemistry, Biology and Physics
Volume.7, Number 3;July-September, 2022;
ISSN: 2836-9572| Impact Factor: 6.62
https://zapjournals.com/Journals/index.php/gjcbp
Published By: Zendo Academic Publishing

A MATHEMATICAL ODYSSEY THROUGH JUPITER'S GREAT RED SPOT DYNAMICS

Elchin Rasulov Ahmedov[1], Lala Aziza Aliyeva[2]

Article Info
Keywords: Self-organization
Astrophysical modeling
Structure formation
Gravitational gas dynamics
Conservation laws

Abstract
Self-organization, characterized by the ordered temporal and spatial evolution of events and structures, is a fundamental phenomenon in the cosmos. Theoretical studies of structure formation in the Universe heavily rely on modeling, as it often represents the only viable approach to validate theoretical predictions, particularly in the context of gravitational gas dynamics with its inherent nonlinearity. This holds true for the intricate processes involved in the development of hierarchical structures, entailing complex alterations in spatial geometry. Consequently, modeling has yielded a vast array of profound astrophysical insights and discoveries. A pivotal aspect of modeling gas dynamics and hydrodynamics is the mathematical representation of the fundamental conservation laws governing continuous media, encompassing mass, momentum, and total energy.

1. Introduction

The phenomenon of self-organization consists of an occurrence of a series of ordered in time and space of events and structures. Consequently, modeling plays a more significant role in theoretical studies of the processes of structure formation in the Universe. Frequently, modeling is the only possible way to confirm by calculations of theoretical predictions in the highly nonlinear regime of gravitational gas dynamics. It is partially correct for the processes of formation of a hierarchical structure with the complex of changes in spatial geometry. Therefore, the list of the most significant founded astrophysical properties and discoveries based on modeling is very long. The meaning of modeling gas dynamics and hydrodynamics is a mathematical expression of the main conservation laws for a continuous medium: mass, momentum, total energy.

In applications of problems of hydrodynamics, it is often necessary to consider additional physical factors, such as heat transfer, burning, gas ionization, the presence of electromagnetic fields, etc. These details lead to the necessity to input new terms into the equations and to include additional equations in the system. Significant distortion is in problems, where extensive deformations and relative displacements of the fluid appear when turbulence occurs.

[1] Department of Planets and small celestial bodies of the Shamakhy Astrophysical Observatory, Azerbaijan
[2] National Academy of Sciences, city Shamakha , Azerbaijan

It should be noted that within each of the approaches of planetary study, there is a wide variety of implementations, each of which has its advantages in its field of application. During the entire time of development of modeling methods, the comparative analysis of solutions of typical astrophysical problems by using various approaches was carried out. The development in this area of modeling is at the stage of creating more detailed models considering an increasing number of physical processes. Therefore, it is appropriate to express every detail of the considered process-planet by using the corresponding equations. In the present paper, we will consider the dynamics of the GRS in the atmosphere of Jupiter. The idea was established that the GRS - long-lived free vortex in the atmosphere of Jupiter according to observations of Voyager-1 in 1979. A vortex of this size and corresponding mass, in the form of a Red Spot, can live for thousands of years. According to observations, the motion of parts over the surface of the Spot has a vortex nature. A large number of hypotheses appeared based on the development of the vortex idea. Although the theory of hydrodynamics is the main law of motion and rotational processes of the GRS, simultaneously, this rotation is consistent synchronously with the image of Jupiter. However, only some researchers have used the theory of hydrodynamics and gas dynamics to describe the dynamic structures of the atmosphere of Jupiter, including the dynamics of the GRS. It is known that the GRS is completely immersed in the usual bright lower tropical zone. GRS sizes vary over time, and currently, its length and width are 26,200 and 13,800 km, respectively. Herewith, several dark filaments connected with the circulating flow interacted with the perimeter of the GRS, and some of them were joined to the outer edge and rotated in the vortex of the GRS in a counterclockwise direction (see [9]). The joined dark spot almost is not showed the 1/r dependence typical for a simple vortex (Fig. b, c, e, f). It should be noted that the results of spacecraft and ground-based observations, which are noted in detail in work [9], shows that GRS surrounded by a bright ring seems to be formed from covering cyclones, and in turn, anticyclones due to the formation of bright ovals shifting inside the vortex flow by completed motions (fig. b, c, e). It is appropriate to attempt to describe this process by using the mathematical model corresponding to the adequate partial derivative equation. Both by physical and mathematical formulations of the GRS motion, in the sense of the sources, are ovals, which, in turn, disappear as interval (intermediate) processes. To complete the justification satisfying the metrological processes of the GRS, although its continuation is identical to the anticyclone system, the sources of the main area is high pressure which forming these procedures (see also [9]), the perturbation of which has been observed for the first time (see [5]). In other words, special attention should be paid to the circulating flow of the southern tropical zone, and then the disturbance of the factor of Jupiter could be interpreted from a mathematical point of view as a description of partial derivative equations. It should be noted that the mathematical model of the process - phenomenon periodically recurring as long as the GRS exists. Namely, during the drifting of the center of the GRS, the Coriolis force plays a significant role, which also ensures the existence of a rotation band (see also [9]). These bands are subjected to the laws of motion of fluid and gas, which are described by using partial derivative equations. In order to characterize the GRS, we need to study the possible motions, and it is necessary to know the properties of fluid particles flow inside and around these parts, also the velocity of the shift of band and spots (see [9]). Probably, this dynamic character of the atmosphere of Jupiter will be interesting in the mathematical meaning to adequate models described by the equations of motion in fluid and gas. In our opinion, despite the fact that a comparatively large number of works are devoted to the chemical composition of the atmosphere, including the observation of some details interacting with the GRS, turned out that the motion and rotational properties had been studied very little based on considerations of mathematical modeling. Consequently, the sequential processes of motion and dynamics of the GRS are comprehensively considered, which described by using partial differential equations.

2. Theoretical justification of the dynamics of the GRS and its fine internal structure.

As is known (see [1-8, 10] and therein) , according to images of the spacecraft "Voyager, the atmosphere of Jupiter is characterized by 12 (and more) zonal jet flows and about 80 vortexes, the largest of which are the Great Red Spot and three White Ovals, which formed in the 1930s, and then merged in 2000 into one White Oval, although the merger of the White Ovals in 1997-2000 was unexpected after more than half a century of existence. However, it is assumed that the disappearance of the White Ovals was not an isolated event but was part of a recurring climatic cycle, which still causes the disappearance of most of Jupiter's vortexes (see [1,2,3,4]). Hot spots are related to three Galilean satellites: Io, Europa, and Ganymede. They arise due to the rotating plasma is slowed down near the satellites. The brightest spots belong to Io since this satellite is the main supplier of plasma, but Europa and Ganymede spots are much weak. Bright spots inside the main rings, appearing from time to time, as considered, are related to the interaction of the magnetosphere and the solar wind (see [9]). Previously considered that many different vortex structures are observed in the atmosphere of Jupiter. As a rule, the pressure in the center of the vortex is higher than in the surrounding area, and the hurricanes themselves from the west are "bound" by disturbances with low pressure. According to Philip Marcus, now Jupiter is in the middle of the process, despite the fact that there is practically no changes of seasons on Jupiter (the axis of the planet is almost perpendicular to the orbital plane), the natural circulation of gas from layer to layer and from belt to belt currently carrying heat to the equator, lowering the temperature near the south pole of Jupiter. At the equator of the planet will be warmer, and at the South Pole, opposite, it will be colder. As a result of the numerical modeling by the author, it is concluded that the disappearance of vortices leads to global temperature changes which destabilizing the atmosphere and causing the formation of new vortices. After formation, large vortexes are destroyed due to turbulence for 60 years (fig e, d, f), (this corresponds to the observations of White Ovals (fig b) - until they disappear after which the cycle already exists. The issues on the long existence of the GRS. Many processes are capable of scattering atmospheric vortexes like the Great Red Spot. Turbulence and atmospheric waves in the Red Spot area absorb the energy of its winds. The vortex loses energy by radiating heat. It should be noted that the absorption of smaller vortexes by the Great Red Spot may be one of the mechanisms for maintaining its life and explains the long century of the largest atmospheric formation of the solar system. However, modern models show that this is not enough. Three-dimensional models taking into account both horizontal and vertical gas flows show that when Spot is losing energy, temperature difference arises, as a result of which hot gas from the lower layers of the atmosphere flows (vertically) into the GRS, which makes it possible to recover some of the lost energy. Therefore, there is a "feeding" of the Red Spot of Jupiter see [5]. As it turned out, vertical motion is the key to the "long life" of the Great Red Spot. The model also indicates the existence of a radial flow that "pulls" wind out of high-rate flows and again directs them to the center of the vortex.

2. Mathematical description of the rotational details and motion process for the dynamics of the GRS on Jupiter.

In this paragraph, we will consider the motion processes and rotational fluid, gas particles as a concentric type of ellipse (or almost like a circle) with thin layers, incompressible, almost inviscid fluid. Since the absence of external forces in the case of an inviscid fluid of motion is described by the following Euler equation:

$$div\vec{V} = 0 \quad (1),$$

$$\frac{\partial \vec{V}}{\partial t} + (\vec{V},\nabla)\vec{V} = -\frac{1}{\rho}\, gradP + \vec{F} \quad (2),$$

□*t* □

Here, in the general case, equation (1) is a mathematical notation of the incompressibility condition, equation (2)

□

□*V*

is the equation of motion for a unit mass, which in its left part consists of the acceleration of motion *W* □___

□*t*

1 □

and particles, hydrodynamic (□ _ *gradP*)and external *F* (as the Coriolis force). Then assume that the fluid is

□

□

in a potential force field, i.e. external force affecting on it *F* (Coriolis, gravitational, and other possible) has the

□

potential U: *F* □ *gradU* . For example, a fluid located in a gravity field and directed along the Z-axis, in this case,

U = -gZ could be taken since the GRS is projected on the plane (x, y, 0). Then, it turns out; the theorem is valid according to the circulation field Γ, the velocity vector on arbitrary closed "fluid» (or "connected particles of a gas band"). More precisely, a closed line consisting of the same fluid particles, under fixing any contour in space, is a fluid contour, even, an ellipse contour, the axes of which are located on the plane (x, y, 0) and remains constant in

*d*Γ *d*

the processes of motion: dt □ *dt* □$_L$(*V*,*dx*) □ 0.

Accordingly, the motion is focused on rest, then the circulation on arbitrarily closed fluid is identically equal to

□

zero. Therefore, by the Stokes formula, we have □(*V*,*dx*) □□(*rotV*,*n*)*ds* , where S- the surface of a thin layer

L *S*

□

of fluid is stretched by the contour L, and it can be assumed from arbitrariness that *rotV* □ 0.It should be noted

□

that the vortex is expressed □□ *rotV*,which determines the angular velocity of rotation of the elementary

□ □

volume of the fluid .If *rotV* □ 0, then rotation is absent. However, in the center of the GRS *rotV* □□,where

□

□is very small (or□□0).Therefore, if□□0, then*rotV* □0, under which this condition is a necessary and

□ sufficient condition for the potentiality of the velocity of field. Therefore, wecanconsider *rotV* □□, under

□□0 condition, quasi-potentiality. Then for the existence of a scalar function □(*x*,*t*) velocity potential, could

□ □ be approximately applied in case *rotV* □0, and, that*V* □ *grad*(□(*x*,*t*)). To check the role of equation (2)

in determining the dependence of the function □(*x*,*t*) on timeand determined pressure P, we must accept,

□ □ □ □ □ ^{2} condition conducting*rotV* □□,at□□ 0.Then from vector analysis we have[*V*,□]*V*

□[□,*V*]□ *grad*□__ , 2

□ □

where□□ *V* velocity vector. Since*rotV* □0, then□□0, consequently, we can replace the following equation

grad(□□□□2 □ *P* □*U*) □ 0.Hence, we obtain Cauchy's integral formula □□□□2 □*P*□*U*□□(*t*), where

□*t* 2 □ □*t* 2 □

□(*t*)□time function, furthermore, this replaces the equation of motion (2).The topography of the "bottom" is derived from three magnitudes obtained from the velocity data - integral of Bernoulli, kinetic energy per mass unit, and absolute vortex [5]. All of them are functions of the horizontal position within the corresponding vortex. Away from the vortex, the potential vortex is calculated depending on the latitude, from the observed zonal velocity of the cloud tops, and obtained topography of the "bottom". The results show that the deep atmosphere is in differential motion and that the potential vortex gradient varies with latitude. It should be concluded that almost at the center of the GRS, the velocity field does not depend on time (i.e., time is constant).

□2 *P*

Then only near of the center of GRS the solution is □ □*U*□*const* and this is the established motion of

2 □

the rotational process of the GRS. Consequently, this condition connects the magnitude of the velocity with pressure, and the potential of the forces *U* , as in ordinary problems, is already known. Since the motions on the plane (x, y, 0) are flat, the level lines □□*const* are the vector of the lines of the velocities field, called the

□□ □□ □□ □□ □*x* □*y* □*y* □*x*

stream function, and connects with the equations □ , □□ . If we take into account the fact that the flow of fluid motion and gas particles is steady-state symmetric relative to the x-axis, and the distance to this axis is denoted by y, and the corresponding coordinates of the velocity vector as V_x,V_y.In this case, the stream function is *y* □□□□, *y* □□□□.However, the motion takes place with given vortices and leads to the

□*x* □*y* □*y* □*x*

well-known identity *rot*(*grad*□) □ 0.So the motion potential is replaced near the center of the GRS on the

□ □2 *P*

given vortices□□ *rotV*, summarized ovals lead to the so-called Lamb function *H*□ □ □*U* where 2 □

□

□□ |*V*|, P-pressure,□-density. Then, applying this expression to equation (2), we obtain

□

□*V* □ □ □

□(*V*,□)*V* □□*gradH*.Then in the case of plane motion, the vector of vortices has the form □□ *rotV* □ □*t*

(□V_y □□V_x)□*k* , which is directed perpendicular to the plane flow, is quite characterized by the scalar quantity

□*x* □*y*

□□□V_y □□V_x.In the unperturbed case, when the Coriolis force gradually jumps from a quasi-laminar

□*x* □*y*

transition to a turbulent one during many years, and the period of jumps, depending on the angular velocity could

□ □

take the form:□□□*k*,in which this is taken into account in the Lamb equation, then □V_y □□*H* ,□V_x □□□*H* ,and the lines correspond to the trajectory of the moving particles. In this case, if the

□*x* □*y*

stream functions □,then □□□□ □□□□□ □ 0.From this it is clear that □□*const*,□□□(□).A similar

□*x* □*y* □*y* □*x*

situation exists in the case symmetric axis, vortices and characterized by the scalar quantity □.Then

□(□)□□ □ □(□)□□ □ *(□)□some function.

0,from which it is seen that □

□x y □y □y y □x

To formulate the problem bordering with the condition, at t> 0 solid fixed boundary near the center of the GRS with a thin layer is chosen. However, due to the dependence on a combination of several factors, the problem could be reduced to the definition and solution of the so-called problem with moving boundaries or a problem with unknown boundaries. Since the motion on fluid circuits around the GRS refers to the approximate, so-called, □ quasi-laminar, then under the condition of incompressibility, $divV$ □0at□□ 0.Therefore, the equations of motion, which are described for a sufficient distance from the center of the GRS, could be replaced by a more general equation

□

□V □

___□div(□V) □ 0 (3),

□t

□

□V □ □ 1

___□(V,□)V □□ _ gradP (4),

□t □

Since diffusion-convective reaction processes are inevitable, and then thermodynamic equations take place dS □S

— □□(V,□)S □ 0.This shows that there is no heat exchange between the particles. This condition means

dt □t

dS

that the entropies S are constant, consequently ___□ 0.Consequently, the equation of state for fluid with very

dt

S

_

P □ P(□,S) , for ideal gas P □ t□□□,□□ ch .At constant entropy, the Bernoulli low viscosity has the form

c□

□2 $c2$

equation takes place — □ ——— □ $const$ Consequently, it turns out that□ the rate of a moving gas is

2 □□1

incompressible and fundamentally differs from the motion of the ideal gas. After simple transformations, the (1□□$_2$2)□1□1 . This ratio could be relative density of the gas is expressed by the following formula:□□□$_0$ □$_m$ considered as the Bernoulli integral. As a result, the rates of motion of the pressure of the fluid, the density of the gas are found easily. The vortex is losing energy, which is of great interest.

It should be noted that there are viscosity and an incompressibility condition around and near the center of the

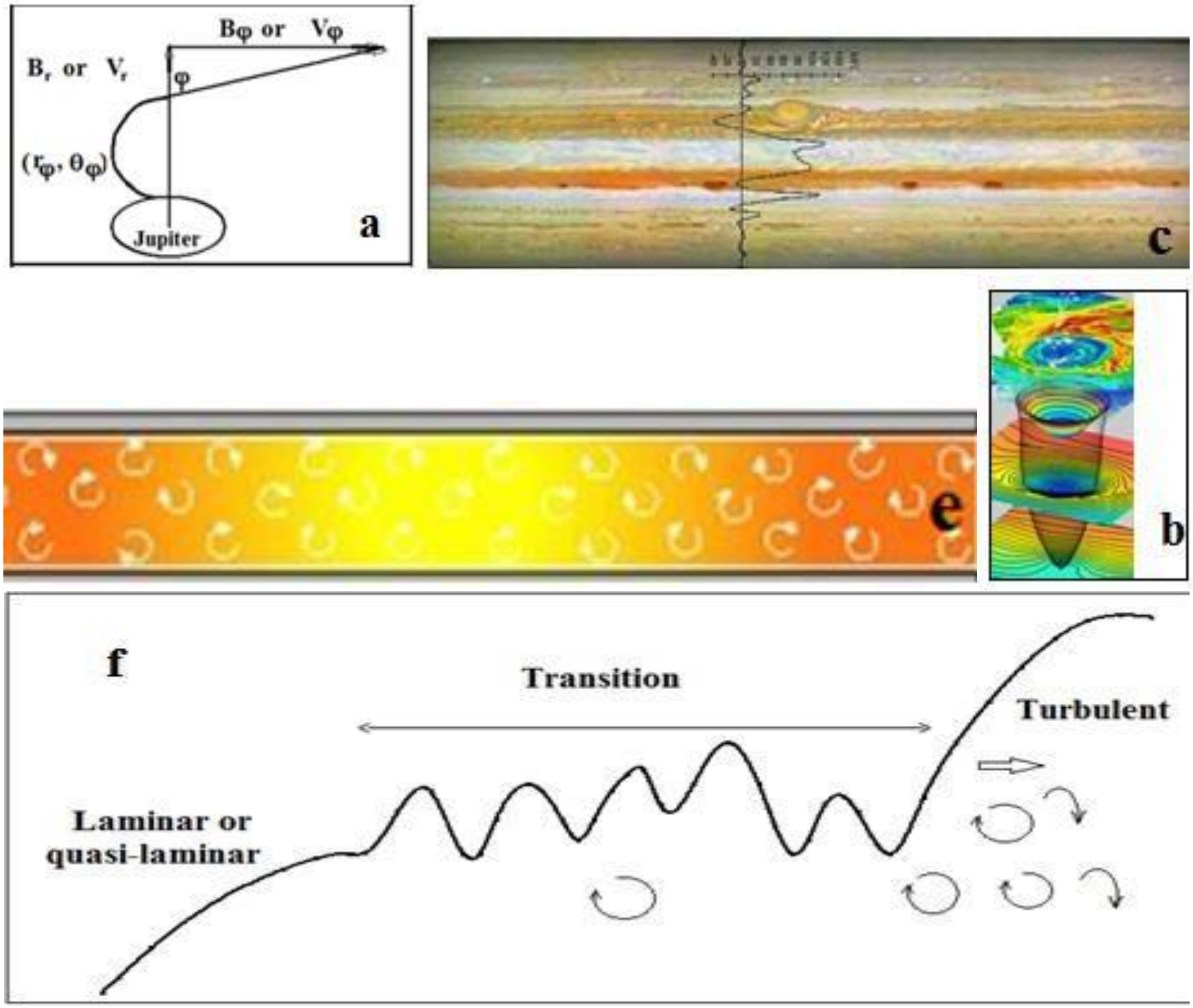
□u □v □x □y □u □u □u 1 □P □^{2}u

GRS, therefore, the equation of motion with viscous terms is described as follows: □ □ 0,

— □^{u} — □^{v} — □□ —— □□ ——

□t □x □y □□x □2x (here, u, v -unknown, P □ $P(x,t)$- gas potential. These systems in their own domain of definition could be approximated for numerical realization. The main source of the GRS dynamics in the atmosphere of Jupiter, interacting with the magnetosphere and the solar wind, forms a spiral

$r = a\varphi$, i.e. is called Parker's spiral (see fig. a, b) and has an equation: $r - r_0 = -(v/\Omega)(\varphi - \varphi_0)$,



Where, $\tan\psi = -\frac{B_\varphi}{B_r} = \frac{v_\varphi}{v_r} = -\frac{\Omega(r - r_0)}{v_r}$. It can be seen from the diagram that the motion of gas and fluid on the GRS is divided into three processes that connect the laminar (or approximate, so-called quasi-laminar) with the transient flow by ovals on turbulent ones. In our opinion, these processes are justified for the first time. Based on this, the application of the theory of hydrodynamics confirms the reliability of such motion processes and adapts to the results of previous observations.

4. Conclusion.

The considered mathematical interpretation of the dynamics of the GRS on Jupiter with all the details was checked by experimental observations. The dynamics of the GRS on Jupiter as a whole, the processes of motion by using a non-classical approach, namely, the flow from laminar (in this case, the peculiarities of the new flow, the so-called quasi-laminar, are the main approach of non-classical application), as well as the transitional flow to turbulent motion have been investigated. Besides, diffusion-convective reaction processes and their descriptive equations, related to the global dynamics of the GRS on Jupiter, were considered.

Moreover, the considered non-classical model also includes the process of the existence of a radial flow, which "pulls" winds from high-rate flows and again directs them to the center of the vortex.

At the same time, it was found in this observation that the source of dynamics of the GRS in the atmosphere of Jupiter, interacting with the magnetosphere and the solar wind, forms a Parker spiral. Thus, applying the theory of hydrodynamics in the non-classical approach complements the choice of the theory of a fixed point with parameters (epsilon) to restore the fulfillment of the condition: vortices, incompressibility of the fluid, rotation of the elementary volume of the fluid, as well as for turbulence and atmospheric waves.

5. References

Appleby*, J.; Orton, G.; Martonchik, J.; Aumann, H.* Properties of NH3 Gas and Ice in the Jovian Atmosphere from Investigation of the Far Infrared Spectrum.//Bulletin of the American Astronomical Society.1981.Vol. 13.P.736

*MarcusP.S.*ПрогнозглобальныхклиматическихизмененийнаЮпитере, NATUREv 428

Choi D. S. , Showman A. P., Vasavada A. R., Simon-Miller A. A. Meteorology of Jupiter's Equatorial Hot Spots and Plumes from Cassini //Icarus.V.223.Issue 2. April 2013. P.832-843 55

Сурдин.Г.М.:Солнечнаясистема :ФИЗМАТЛИТ,2009,(http://spacehorizon.ru/articles/20)

Мороз В.И. Физикапланет, М. «Наука», 1967. 496 С 24

Морозов А. 2013. Простая физика Волны Россби и природа циклонов и антициклонов на планетах. http://moralg.livejournal.com/210356.html *Bhardwaj Anil and Marykutty Michael* Io-Jupiter system: a unique case of moon-planet interaction. //JupiterIo.pdf

Bagenal, F.; Dowling, T. E Jupiter: The planet, satellites, and magnetosphere

AdaletAtai and MahammadA. Nurmammadov .Influences the factors of Cyclones, Anticyclones, Circulation and Rotation to the Steady Dynamic of Great Red Spot of Jupiter. Journal of Natural Sciences .Vol. 8. No.2 December 2020, pp. 16-19 Published by American Research Institute for Policy Development ВладимирВдовиченко,ГалинаКириенко.АтмосфераЮпитера. БольшоеКрасноеПятно.. www.ljubljuknigi.ru , ISBN: 978-3-659-63264-8 , Lambert Academic Publishing , Germany, 2014