

# HEATING UP THE GROUND: TEMPERATURE-INCLUSIVE CONSOLIDATION ANALYSIS OF GRANULAR COLUMN-REINFORCED COMPOSITE FOUNDATIONS UNDER VARIABLE LOAD

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## Abstract

Granular columns have emerged as a prevalent and effective solution for the treatment of soft soil foundations, owing to their remarkable strength and superior water permeability. While analytical solutions, such as those based on Barron's equal strain condition, have been developed for scenarios involving immediate loading, practical engineering applications rarely involve instantaneous load application. In response to the real-world complexities, this study delves into the impact of variable loads on granular column-reinforced composite foundations. The foundational work laid by Xie [2] and Tang [3], utilizing Barron's equal strain condition for immediate loading scenarios, forms the basis of existing analytical solutions. However, the inherent variations in load application in practical engineering necessitate a more nuanced approach. Existing research, as evidenced by [4-9], has addressed the consolidation theory of composite foundations, specifically considering the influence of variable loads. This paper builds upon this foundation, aiming to provide a comprehensive understanding of the interplay between granular columns and varying loads in the context of composite ground. The study incorporates theoretical analyses and practical insights to elucidate the behavior and performance of granular column-reinforced composite foundations under variable loading conditions. By exploring the consolidation theory within the realm of variable loads, the research aims to contribute essential insights for optimizing the treatment of soft soil foundations in practical engineering applications. Through a synthesis of theoretical advancements and empirical findings, the study offers a valuable resource for engineers and researchers seeking to enhance the efficacy of granular column-based solutions in addressing the challenges posed by soft soil foundations.

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## Introduction

Granular columns are widely used in soft soil foundation treatment because of their high strength and good water permeability. Analytical solutions using Barron's [1] equal strain condition for the case of immediate loading were given by Xie [2] and Tang [3]. In practical engineering, loads are not applied instantaneously. Therefore, it is necessary to consider variable loads in the study of composite ground with granular columns. [4-9] analyzed the consolidation theory of composite foundation under the influence of variable load. With the development of consolidation theory, temperature effects are gradually being applied. Wu [10] derived the one-dimensional thermal consolidation equation of saturated soils and its analytical solution. Wang et al [11] showed that the rate of soil consolidation is accelerated in case of temperature increase. Deng et al [12] investigated the rationality of the theory of soft soil foundation consolidation by considering the temperature effect. In the above theoretical studies, the consolidation deformation of granular columns is not considered. Lu [13] and Zhao [14] considered the consolidation deformation of piles for this point, and obtained the consolidation solution of composite ground. However, in their study, they did not consider the temperature effect, which affects the rate of consolidation in practical engineering. Therefore, this paper establishes a computational model of composite ground with granular column under consideration of temperature change and single-stage loading, derives an analytical solution of composite ground with granular column under consideration of temperature effect and single stage loading, conducts degradation analysis of this analytical solution and compares it with the existing solution to verify its reasonableness and correctness. Finally, the analysis of the consolidation characteristics of composite ground with granular columns is summarized.

### 1. Establishment of Calculation Sketch and Basic Assumptions.

Figure 1 shows a simplified model of the composite ground with granular columns considering temperature effect and variable load.  $H$  is the thickness of the soft soil layer;  $q(t)$  is the variable load;

$r_w, r_s, r_e$  are the radius of the granular column, the radius of the smear zone and the radius of the influence zone;  $k_w$  is the vertical permeability coefficient of the vertical drain;  $k_h, k_s$  are the horizontal permeability coefficients of soil, the horizontal permeability coefficients of remolded soil respectively;  $E_s, E_w$  are the compression modulus of the natural soil and granular column, respectively;  $u_w$  are the pore pressure within the vertical drain and soil at any point, respectively;  $u_r, u_s$  are the pore pressure within the natural soil zone and smear zone at any point, respectively;  $z, r$  are the vertical and radial coordinates, respectively, assuming that the drainage conditions of the composite foundation are permeable at the top and impermeable at the bottom.

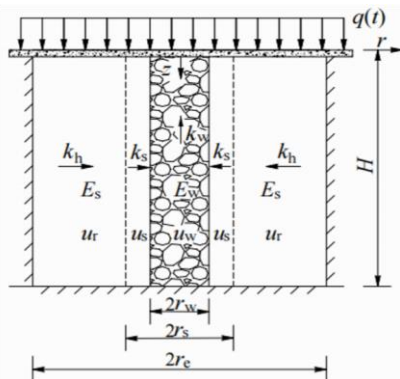


Figure 1: Computing model.

In the derivation of this paper, the following assumptions are made about the model.

- (1) The equal strain condition holds; and only radial water flow within the soil body is considered while vertical water flow is neglected.
- (2) Neglecting the radial seepage in the vertical drain.
- (3) Temperature changes will only affect the permeability of the soil. The relationship between temperature and permeability coefficient is derived from the literature [11] as follows:

$$k_T = (aT + b)k_R \quad (1)$$

In the previously mentioned equation,  $k_T$  and  $k_R$  are the coefficients of permeability at the experimental temperature and room temperature, respectively;  $a$  and  $b$  are all constant.

- (4) Considering the radial consolidation of the soil around the pile, the difference between the pore water inflow and the outflow pile is equal to the change of the pile volume.

$$\frac{\partial}{\partial z} \left( \frac{2k_s T}{E_s} \frac{\partial u}{\partial r} \right)_{r=r_w} - \frac{\partial}{\partial r} \left( \frac{2k_w}{E_w} \frac{\partial u}{\partial r} \right)_{r=r_w} = \frac{\partial}{\partial t} \left( \frac{V}{V_0} \right) \quad (2)$$

$$\frac{\partial}{\partial z} \left( \frac{2k_s T}{E_s} \frac{\partial u}{\partial r} \right)_{r=r_w} - \frac{\partial}{\partial r} \left( \frac{2k_w}{E_w} \frac{\partial u}{\partial r} \right)_{r=r_w} = \frac{\partial}{\partial t} \left( \frac{V}{V_0} \right)$$

## 2. Derivation of the consolidation equation

Based on the equilibrium condition and the assumption of equal strain:

$$\pi(r_e^2 - r_w^2) \sigma_s + \pi r_w^2 \sigma_w = \pi r_e^2 q(t) \quad (3)$$

$$\frac{\partial}{\partial z} \left( \frac{2k_s T}{E_s} \frac{\partial u}{\partial r} \right)_{r=r_w} - \frac{\partial}{\partial r} \left( \frac{2k_w}{E_w} \frac{\partial u}{\partial r} \right)_{r=r_w} = \frac{\partial}{\partial t} \left( \frac{V}{V_0} \right) \quad (4)$$

Where  $\sigma_s, \sigma_w$  are the average stresses in the soil, granular column, respectively;  $\epsilon_v$  is the volume strain in a composite foundation;  $u$  are the average pore water pressure in the foundation soil at any depth; the expressions are

$$\frac{\partial \epsilon_v}{\partial z} = \frac{1}{E_s} \frac{\partial u}{\partial z} + \frac{\partial u}{\partial r} \left( \frac{r_w}{r_e} \right)^2 \frac{dq}{dr} \quad (5)$$

$$\frac{\partial}{\partial t} \left( \frac{2k_s T}{E_s} \frac{\partial u}{\partial r} \right)_{r=r_w} - \frac{\partial}{\partial r} \left( \frac{2k_w}{E_w} \frac{\partial u}{\partial r} \right)_{r=r_w} = \frac{\partial}{\partial t} \left( \frac{V}{V_0} \right) \quad (5)$$

Where  $n = r_e/r_w$  is the radius ratio.

Consolidation equation for the soil can be obtained as

$$\frac{k_s T}{E_s} \frac{\partial}{\partial z} \left( \frac{\partial u}{\partial r} \right)_{r=r_w} + \frac{\partial}{\partial r} \left( \frac{2k_w}{E_w} \frac{\partial u}{\partial r} \right)_{r=r_w} = \frac{\partial}{\partial t} \left( \frac{V}{V_0} \right) \quad (6)$$

$$\frac{k_s T}{E_s} \frac{\partial}{\partial z} \left( \frac{\partial u}{\partial r} \right)_{r=r_w} + \frac{\partial}{\partial r} \left( \frac{2k_w}{E_w} \frac{\partial u}{\partial r} \right)_{r=r_w} = \frac{\partial}{\partial t} \left( \frac{V}{V_0} \right) \quad (7)$$

The average pore pressure in the foundation soil at a certain depth of  $z$  is

$$1 \leq r \leq r_e$$

$$u = (r e^2 - r w^2) (r w^2 \square r u_s d r + r s^2 \square r u_r^{dr}) \quad (8)$$

### 3. The solution of the equation

Integrating equation (6) and (7) with respect to  $r$ , and introducing the boundary conditions, the following two equations are obtained,

$$u_s(z, r, t) = \square w \square r e^2 \ln r - r - r w \square \square \partial \square v + u w \quad (9)$$

$$u_r(z, r, t) = \square \square \square w \square r^2 \square r^2 - r s^2 \square \square w \square^2 \square r s^2 - r w^2 \square \square \square \ln r_e - \square + \square r_e \ln s - \square \square u_w \quad (10)$$

$$\square 2 k_{hT} \square r_s \square 2 \square 2 k_{sT} \square^2 \square \square$$

Where  $s = r_s / r_w$ .

Substituting equation (9) and (10) into equation (8), yields,

$$u = + u_w \quad (11) \quad 2 k_{hT} \partial t$$

$$F_a = \square \ln + \square \ln s - \square^2 + \square^2 \square 1 - \square \square 1 - \square^2 \square + \square^2 \square 1 - \square^2 \square \quad (12)$$

$$\square \square s \square k_{sT} \square 4 \square n - 1 \square n - 1 \square k_{sT} \square \square 4 n \square k_{sT} n - 1 \square 4 n \square$$

Based on equation (2), yields,

$$\partial^2 u_w \square \square w \square^2 \partial \square V = - n \quad (13)$$

$$\partial^2 z^2 \square k_w \square \partial t$$

From equation (11) and (13), the following is obtained:

$$\frac{\partial^2 u_w}{\partial z^2} \rho^2 = - (u u_w +) (a T b) \quad (14)$$

According to equation (4) and (9), it follows that

$$\partial^3 u_w \partial^2 u_w \square \partial u_w \square^2 \square d q \quad (15)$$

$$2 + \square (a T + b) \square^2 - \square (1 + F)(a T + b) + \square (1 + F)(a T + b) = 0$$

$$\partial z \partial t \partial z \partial t \partial t \partial t$$

$$1 \square k h E s \square 8 c h \text{ Where } F = n^2, \square h =, \square = F a d e^2 (1 Y F),$$

$$-1 \quad \square W$$

$$2 \quad 8kh n^2$$

$$\square = 2 .$$

Fade kw

According to the homogenization principle and orthogonal relation of trigonometric function series, the following can be obtained:

$$u = \sum_{m=1}^{\infty} \sin \square \int_0^t e^{-\beta_m(t-\tau)} M_m \square H \square_0 d\tau \quad (16)$$

The average degree of consolidation at any time in the composite foundation soil is

$$U(t) = \frac{1}{\sum_{m=1}^{\infty} 2} \int_0^t e^{-\beta_m(t-\tau)} d\tau \quad (17)$$

$$qu = \sum_{m=1}^{\infty} M_m \square_0 d\tau$$

$$kwEs (n^2 - 1 + Y) \square M \square 2$$

$$\gamma^w n^4 \square \square H \square, M = \frac{2m-1}{\square}, m=1,2,3 .$$

$$\text{where } \beta_m = \frac{F r^2 (2 - )}{\square^2 2 k n 1}$$

$$a_e \quad w \quad M \square$$

$$1 + \quad 4 \quad \square \quad \square$$

$$2n k_{hT} \square H \square$$

#### 4. Expression for degree of consolidation under single-grade loading in equal rates

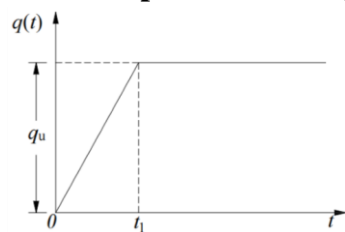


Figure 2: Single-ramp loading.

As shown in Figure 2, the mathematical expression for single-ramp loading is

$$\square q \underline{u}$$

$$\square \quad t, 0 \leq t \leq t_1 \quad q(t) = \square t_1 \quad (18)$$

$$\square \square q_u, t_1 < t$$

Substituting equation (18) into equation (16) and (17) yields, thus, the total average consolidation of a composite foundations can be derived as

$$qu \infty 1 2 \quad \square Mz \quad \square \quad -\square mt$$

$$0 \leq t \leq t_1 : u = \sum \sin \square \quad \square (1 - e^{-\square t}) \quad (19) \quad t_1 \quad m=1 \quad \square m M \quad \square H \quad \square$$

$$U(t) = \sum_{m=1}^{\infty} \frac{1}{2} (1 - e^{-\beta_m t}) \quad (20)$$

$$u = \sum_{m=1}^{\infty} \frac{1}{2} \left[ Mz - e^{-\beta_m(t-t_1)} - e^{-\beta_m t} \right] \sin \frac{\beta_m z}{H} \quad (21)$$

$$U(t) = \sum_{m=1}^{\infty} \frac{1}{2} \left[ \frac{2}{t} - 1 - e^{-\beta_m(t-t_1)} - e^{-\beta_m t} \right] \quad (22)$$

When temperature effects are not considered,  $\beta_m$  can be degraded to the solution of Wang [5] when the vertical seepage in the undisturbed zone is neglected. At this point,  $\beta_m$  is denoted as

$$\beta_m = \frac{2}{H} \sqrt{\frac{k_w}{4k_h} (n^2 - 1 + Y)} \quad (23)$$

Based on the above formula continued degradation, if we take  $t_1 \rightarrow 0$ , then formula (16) and formula (17) can be written as

$$u = \sum_{m=1}^{\infty} \frac{1}{2} e^{-\beta_m t} \sin \frac{\beta_m z}{H} \quad (24)$$

This solution is in the form of the composite ground consolidation solution when the load is applied instantaneously by Zhao [13].

To facilitate further calculation, some parameters are now dimensionless.

$$\begin{aligned} T &= \frac{c t}{Th} = \frac{E k}{4re} \frac{t}{h^2} \\ \Gamma &= \frac{\beta_m t}{4re} = \frac{2}{4re} \frac{t}{h^2} \end{aligned} \quad (25)$$

$$\begin{aligned} &8(n^2 - 1 + Y)(aT + b)M^2 \\ &F_a(n^2 - 1)H^2 \\ &\Gamma_m = 2K_h n^3 (aT + b) M^2 \\ &+ 2 \\ &F_a 2K_w (n^2 - 1) H \end{aligned} \quad (26)$$

## 5. Analysis of Consolidation Properties

In the study of consolidation characteristics of composite ground with granular columns, the radial time factor of the composite ground is taken as the horizontal coordinate and the overall average degree of consolidation is taken as the vertical coordinate. With reference to the actual project, the radius of the granular column is taken as

$r_w=0.25\text{m}$ , the compression modulus of the soil is taken as  $E_s=5\text{MPa}$ , and the vertical permeability coefficient of the pile is taken as  $k_w=6.0\times 10^{-5}\text{m/s}$ .

Figure 3 shows the curves of the effect of different temperatures on the consolidation process of the foundation. It can be seen that as the temperature increases, the rate of foundation consolidation accelerates for the same amount of time and the time required for soil consolidation decreases. Moreover, under the same temperature difference conditions, the higher the temperature, the effect of temperature on the foundation soil becomes less and less.

Figure 4 shows the graphs of the variation of excess pore pressure with time factor at different depths. As can be seen from the figure, when  $T_h < T_{h1}=0.2$ , the load is in the constant loading stage, the excess pore pressure increases with time, and the pore pressure dissipates more slowly with the increase of depth. The excess pore pressure reaches its maximum value at  $T_h=0.2$ . When  $T_h > T_{h1}=0.2$ , the load is in the constant load stage, when the load reaches its maximum value. Due to the different depths of the soil, the pore pressure distribution of the soil is different, the smaller the depth, the smaller the excess pore pressure is, and with the growth of time, the excess pore pressure gradually dissipates.

Figure 5 shows the graph of the effect of different temperatures and loading time on the excess pore pressure. From the figure, it can be seen that in the constant loading stage, the shorter the loading time is, the faster the loading rate is, and the greater the maximum value of excess pore pressure is reached at the end of loading. For the same loading time, the larger the maximum value reached at the end of the constant loading phase, the faster the rate of dissipation of excess pore pressure as the temperature rises. The effect of temperature on excess pore pressure varies as the loading time varies, the longer the time, the greater the effect of temperature on excess pore pressure.

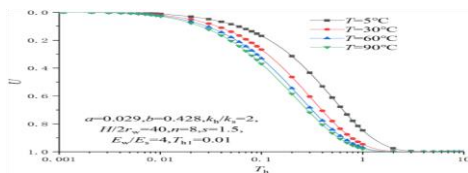


Figure 3: Curves of the effect of different temperatures on the consolidation process of the foundation

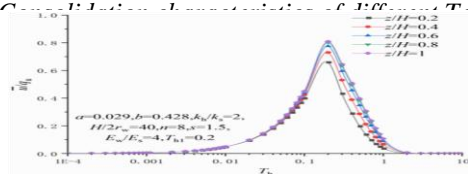


Figure 4: Effect curves of soil depth on the excess pore pressure

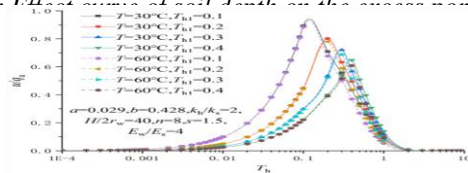


Figure 5: Effect curves of different temperatures and loading times on the excess pore pressure

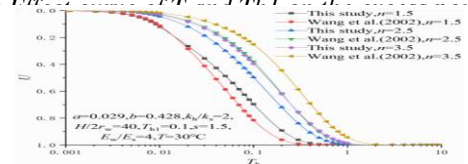


Figure 6: A comparison of the consolidation process of composite ground with and without pile consolidation deformation

Figure 6 shows the plot with the solution of Wang (2002) at different pile diameter ratios. Influence of temperature effect and pile consolidation deformation on the consolidation rate of composite ground not considered in Wang's solution. When the pile diameter ratios are relatively small, the consolidation deformation of the pile body is

considered to slow down the consolidation rate, and the effect of temperature on the consolidation rate is very small. As the pile diameter ratios increase, the effect of pile consolidation deformation on the consolidation rate is considered to be less and less, while the effect of temperature on the consolidation rate gradually increases.

## 6. Conclusion

In this paper, the analytical solution for consolidation with consideration of temperature effect and variable load is given and compared with the existing solution. Based on this solution the properties of temperature and variable load influenced composite ground consolidation of granular columns were analyzed and the following conclusions were obtained:

- (1) The analytical solution for the consolidation of composite ground with granular columns is obtained by considering the temperature change, variable load and pile consolidation deformation. The correctness and reasonableness of this solution are verified by degradation analysis.
- (2) Under the influence of temperature, the consolidation rate of the composite ground of granular columns changes, and the consolidation rate of the foundation accelerates with the increase of temperature, and the higher the temperature is, the influence of the temperature effect on the soil of the foundation becomes smaller and smaller under the condition of the same temperature difference; The longer the loading time, the slower the rate of consolidation of the composite ground.
- (3) As the temperature increases, the greater the maximum value reached at the end of the equal speed loading phase for the same loading time; The longer the loading time, the greater the effect of temperature on the excess pore pressure.

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