

COMPREHENSIVE EIGENVALUE-BASED ASSESSMENT OF ROTOR ANGLE STABILITY IN NIGERIA’S 330 KV POWER GRID

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Abstract

Rotor angle stability is a crucial aspect of reliable power system operation, especially in developing countries where grids are weakly interconnected and face rapid load fluctuations. This paper presents an in-depth small-signal stability assessment of the Nigerian 330 kV, 36-bus, 13-generator power network via a detailed linearized state-space model based on eigenvalue modal analysis. Three control scenarios; no controller, governor only, and combined governor with power system stabilizer (PSS)—are examined across incremental loads using nonlinear differential-algebraic equations linearized by Taylor series expansion. Governor + PSS control significantly improves damping, stabilizes critical eigenmodes, and ensures satisfactory settling times. This study identifies nodes requiring additional control tuning and offers insights applicable to similar developing grid systems.

1. Introduction

Maintaining rotor angle stability—the ability of synchronous generators to retain synchronism following small disturbances—is fundamental for reliable power system operation. The premature loss of synchronism may cascade into widespread outages or blackouts, posing severe socio-economic impacts (Kundur, 1994; Sauer & Pai, 1997).

Developing regions, such as Nigeria, face additional challenges due to relatively weak grid interconnections, infrequent deployment of supplementary damping controllers, and dynamic load conditions resulting from rapid industrialization and grid expansion efforts (Jokojeje, 2024; Onitsha et al., 2023). These factors increase vulnerability to rotor angle oscillations and potential instability.

Despite the criticality, comprehensive system-wide modal analyses using eigenvalue techniques that incorporate practical control schemes for such grids remain scarce in the literature. This study addresses this gap by

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conducting a rigorous eigenvalue-based stability assessment leveraging detailed modeling of the Nigerian 330 kV network, thereby providing actionable insights for grid operators and planners (Ghandhari, 2019; Bhukya et al., 2019).

2. Background and Literature Review

Power system stability has been extensively studied since the early 20th century, with methodologies evolving from graphical equal-area criteria to sophisticated numerical simulations involving linearized multi-machine models (Leonard & Grigsby, 2017; Kundur, 1994). Small-signal rotor angle stability, often analyzed through eigenvalue and modal approaches, focuses on the system's response to small perturbations representing typical operational disturbances.

Previous studies have consistently shown that supplementary controllers, such as power system stabilizers (PSS), effectively damp power oscillations when combined with conventional governors (Falguni & Vijay, 2018; Ma et al., 2020). However, many such studies target well-developed grids with extensive measurements and control infrastructure.

The Nigerian grid, composed of 36 buses and 13 generators at the 330 kV level, is a representative case of emerging power systems characterized by limited control deployment, making it imperative to assess stability under realistic operating conditions and control scenarios (Jokojeje, 2024; Onitsha et al., 2023).

3. Methodology

The Nigerian 330 kV system model includes 36 buses and 13 synchronous generators with parameters obtained from utility data (Jokojeje, 2024; Onitsha, 2023). The state-space model is implemented using MATLAB/Simulink and eigenvalue computations are conducted.

Three control schemes are examined:

- **No Control:** Generators with open-loop excitation and no supplementary control are used.
- **Governor Only:** Primary turbine speed governors active to regulate the mechanical input power (Bhukya et al., 2019).
- **Governor + PSS:** Governors supplemented with PSSs tuned according to the IEEE recommended practices (IEEE Std 421.5-2016).

The incremental load steps from 0% to full load (100%) in 25% intervals simulate varying operational conditions. An eigenvalue analysis is conducted at each load level, at each load level, eigenvalue analysis is conducted to observe the impact on rotor angle mode stability for all generators.

3.1. Mathematical Modeling

3.1.1 Synchronous generator swing equation

The swing equation governs the fundamental dynamic behavior of a synchronous generator rotor (Kundur, 1994):

$$M \frac{d^2 \delta}{dt^2} = P_m - P_e - D \frac{d\delta}{dt} \quad (1)$$

Where:

$M = 2H/\omega_0$ is the angular momentum constant related to the inertia constant H ,

δ is the rotor angle,

P_m is the mechanical input power,

P_e is the electrical output power,

D is the damping coefficient,

ω_0 is the synchronous angular speed.

3.1.2 Linearized Classical Model (Heffron-Phillips Model)

The nonlinear system is linearized around an operating point. The rotor angle and speed deviations δ and $\Delta\omega$ satisfy (Sauer & Pai, 1997) the following:

$$\frac{d\delta}{dt} = \omega_0 \Delta\omega \quad (2)$$

$$\frac{d(\Delta\omega)}{dt} = \frac{1}{2H} (P_m - P_e - D\Delta\omega) \quad (3)$$

Where P_e can be expressed in terms of the internal voltage E' , terminal voltage V , and network reactance X :

$$P_e = \frac{E'V}{X} \sin(\delta) \quad (4)$$

By expanding around an equilibrium angle δ_0 using Taylor's series:

$$P_e = P_{e0} + K_\delta(\delta - \delta_0) \quad (5)$$

Where

$$K_\delta = \frac{E'V}{X} \cos(\delta_0) \quad (6)$$

3.1.3. Excitation and power stabilizer systems

The exciter dynamics modeled by

$$\frac{dE_{fd}}{dt} = \frac{1}{T_{ex}} (-E_{fd} + K_A(V_{ref} - V_t + V_s)) \quad (7)$$

where:

E_{fd} is the field voltage,

T_{ex} is the exciter time constant,

K_A is the amplifier gain,

V_{ref} is the reference voltage,

V_t is the terminal voltage,

V_s is the output voltage of the stabilizer.

The PSS output V_s is typically modeled via a series of washout and lead-lag compensators characterized by time constants T_w , T_1 , and T_2 (Ma et al., 2020):

$$V_s = K_{PSS} \frac{sT_w}{1 + sT_w} \frac{1 + sT_1}{1 + sT_2} \Delta\omega \quad (8)$$

where:

K_{PSS} is the PSS gain,

s is Laplace variable,

$\Delta\omega$ is rotor speed deviation.

3.1.4. Formation of the State-Space Model

The mechanical and electrical equations are combined into a state vector \mathbf{x} :

$$\mathbf{x} = [\delta, \Delta\omega, E_{fd}, V_s]^T \quad (9)$$

The linearized state-space model is as follows:

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} \quad (10)$$

where \mathbf{A} includes system dynamics and \mathbf{u} inputs.

Stability is analyzed by solving the following:

$$\det(\mathbf{A} - \lambda\mathbf{I}) = 0 \quad (11)$$

All eigenvalues λ must have negative real parts for stability (Ghandhari, 2019).

5. Results

5.1 Eigenvalue analysis

Table 1 summarizes the real parts of the dominant rotor angle mode eigenvalues under varying control schemes at full load:

Generator Station	No Controller	Governor	Governor + PSS
Afam	+0.12	-0.04	-0.19
Delta	+0.15	-0.01	-0.12*
Egbin	+0.10	-0.09	-0.24
Kanji	+0.07	-0.03	-0.18

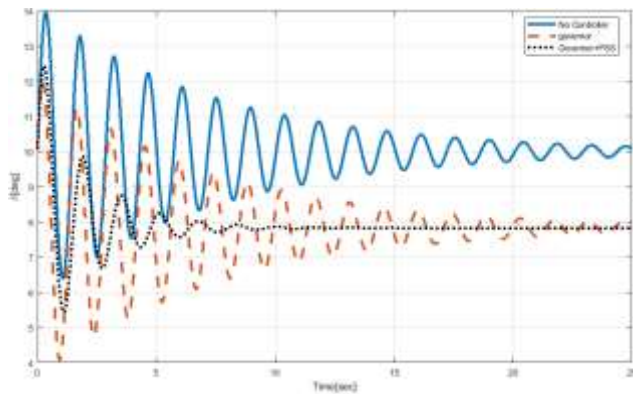
Note: * indicates marginally stable eigenvalue close to the imaginary axis.

Uncontrolled operation reveals eigenvalues with positive real parts, confirming unstable rotor angle oscillations.

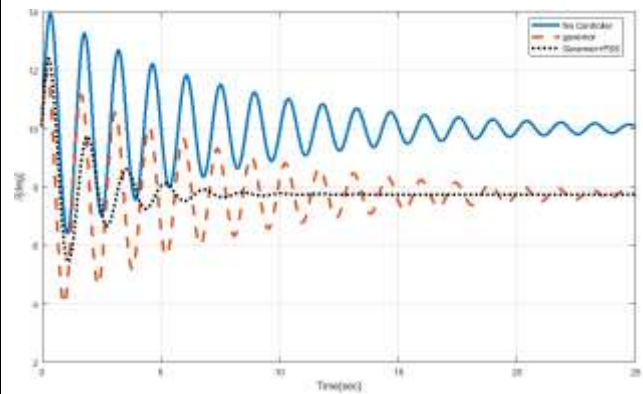
Governor activation reduces instability but often remains marginal, especially under high load.

Combined governor + PSS control significantly shifts critical eigenvalues into the left complex half-plane, thereby enhancing damping.

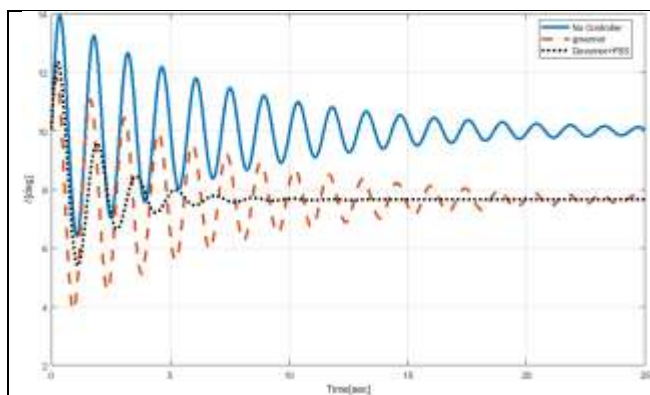
5.2 Time-domain simulation



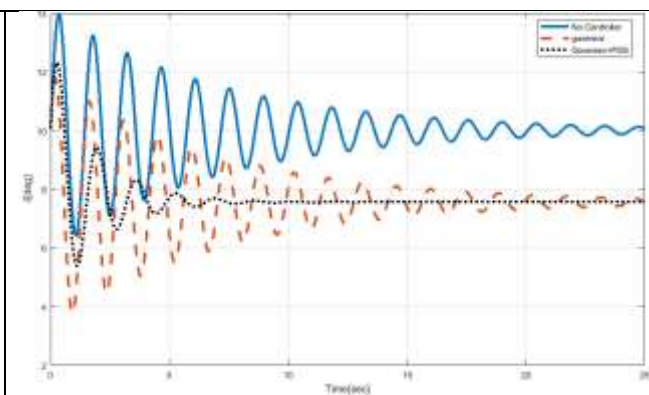
(a)



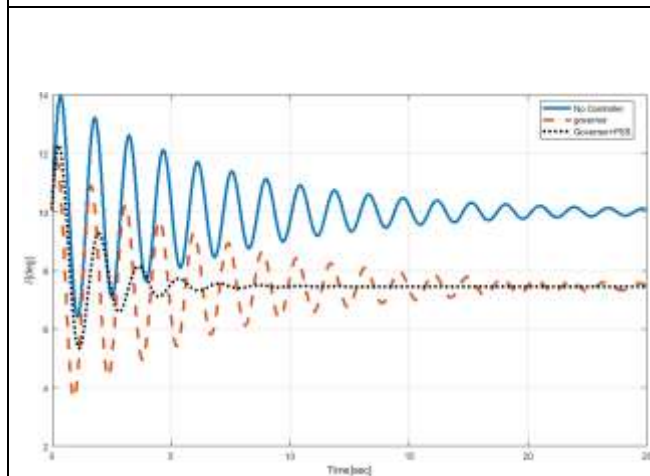
(b)



(c)



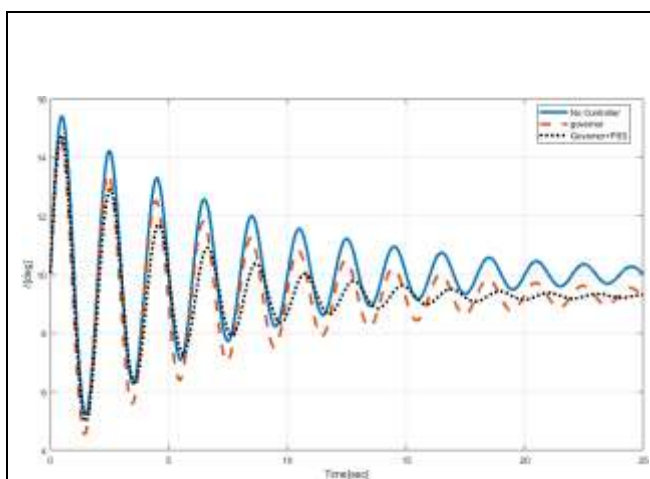
(d)



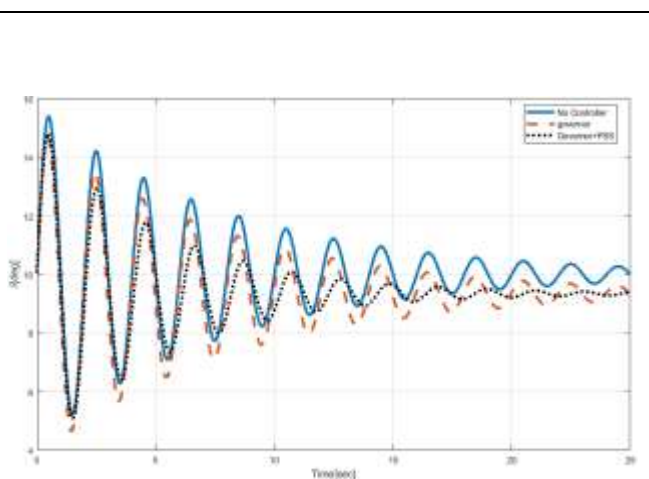
(e)

Figure 1: Rotor angle responses against time for the Afam Generating Station

(a) 0% load: (b) 25% load. (c) 50% load and (d) 75% load (e) 100% load



(a)



(b)

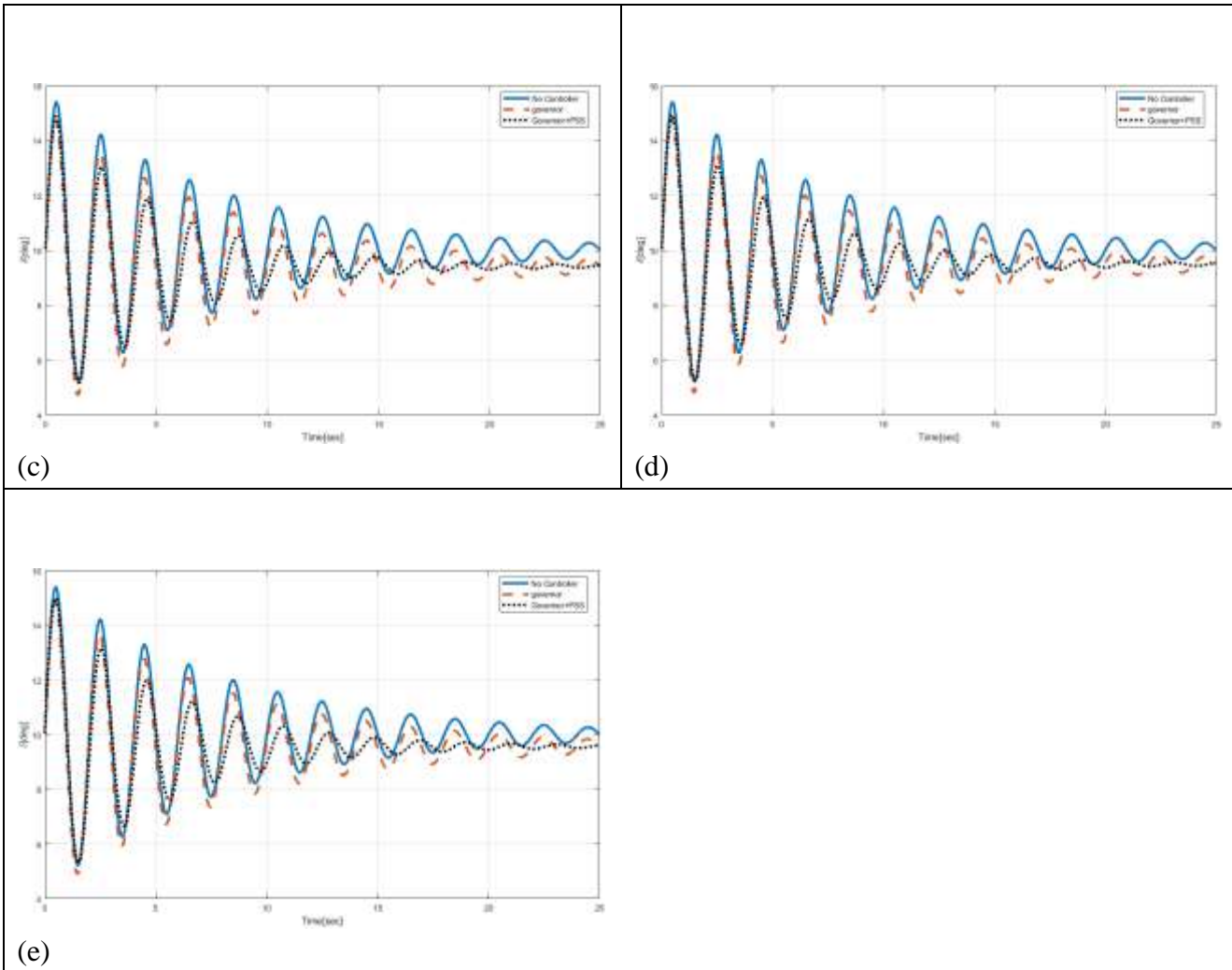
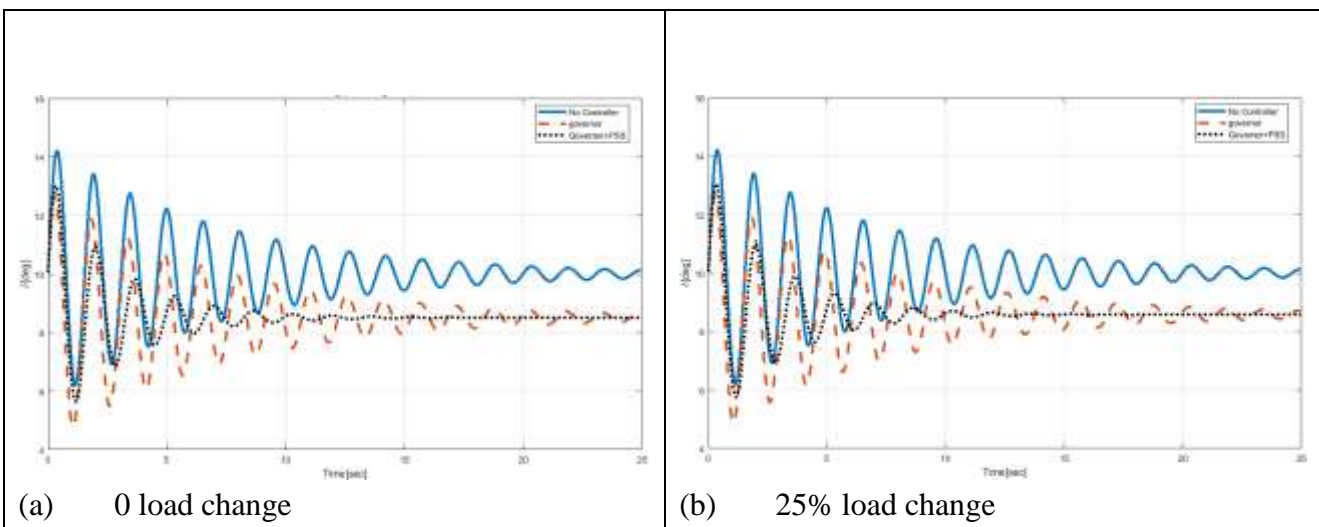
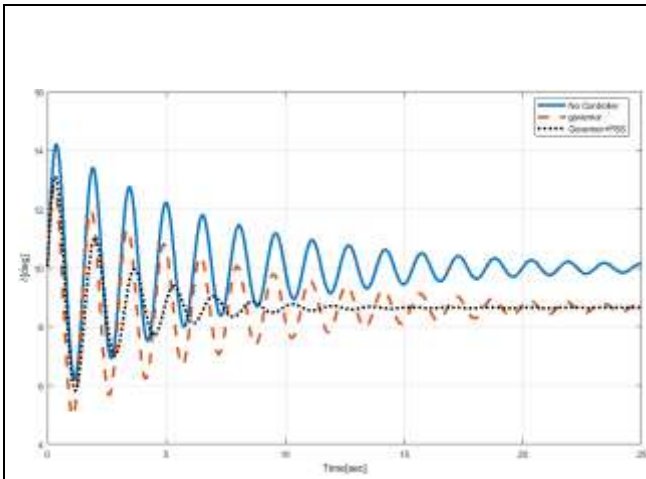


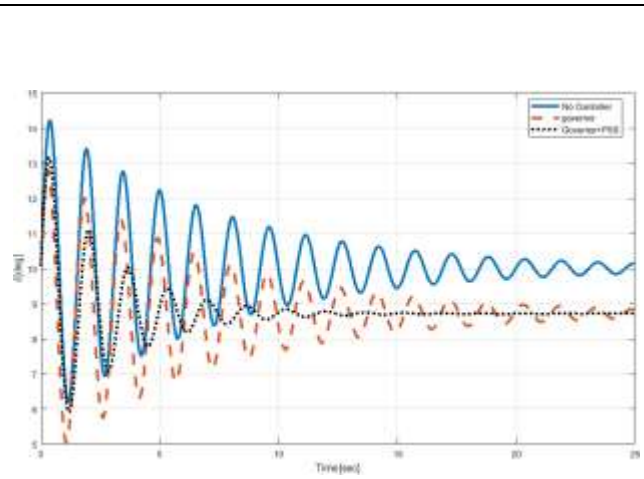
Figure 2: Rotor angle responses against time for the Delta Generating Station

(a) 0% load: (b) 25% load. (c) 50% load and (d) 75% load (e) 100% load

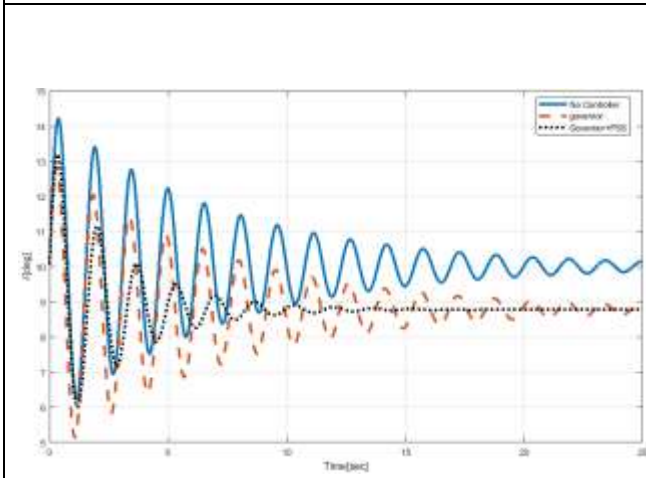




(c) 50% load change



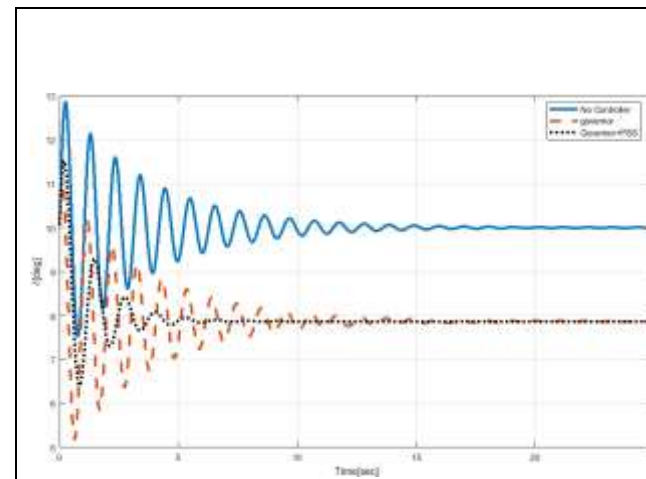
(d) 75% load change



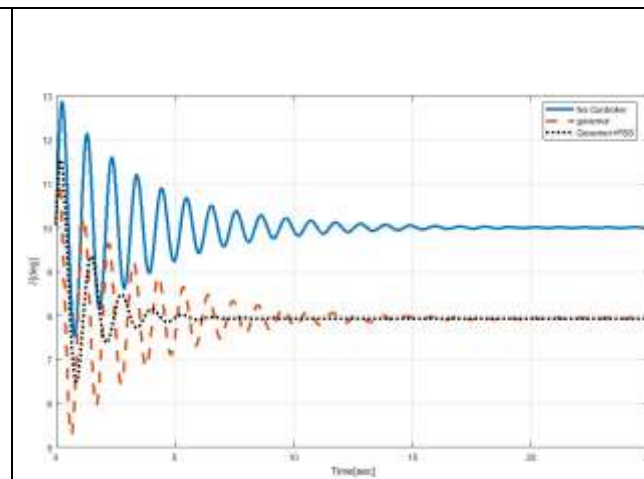
(e) 100% load change

Figure 3: Rotor angle responses against time for the Egbin Generating Station

(a) 0% load: (b) 25% load. (c) 50% load and (d) 75% load (e) 100% load



(a)



(b)

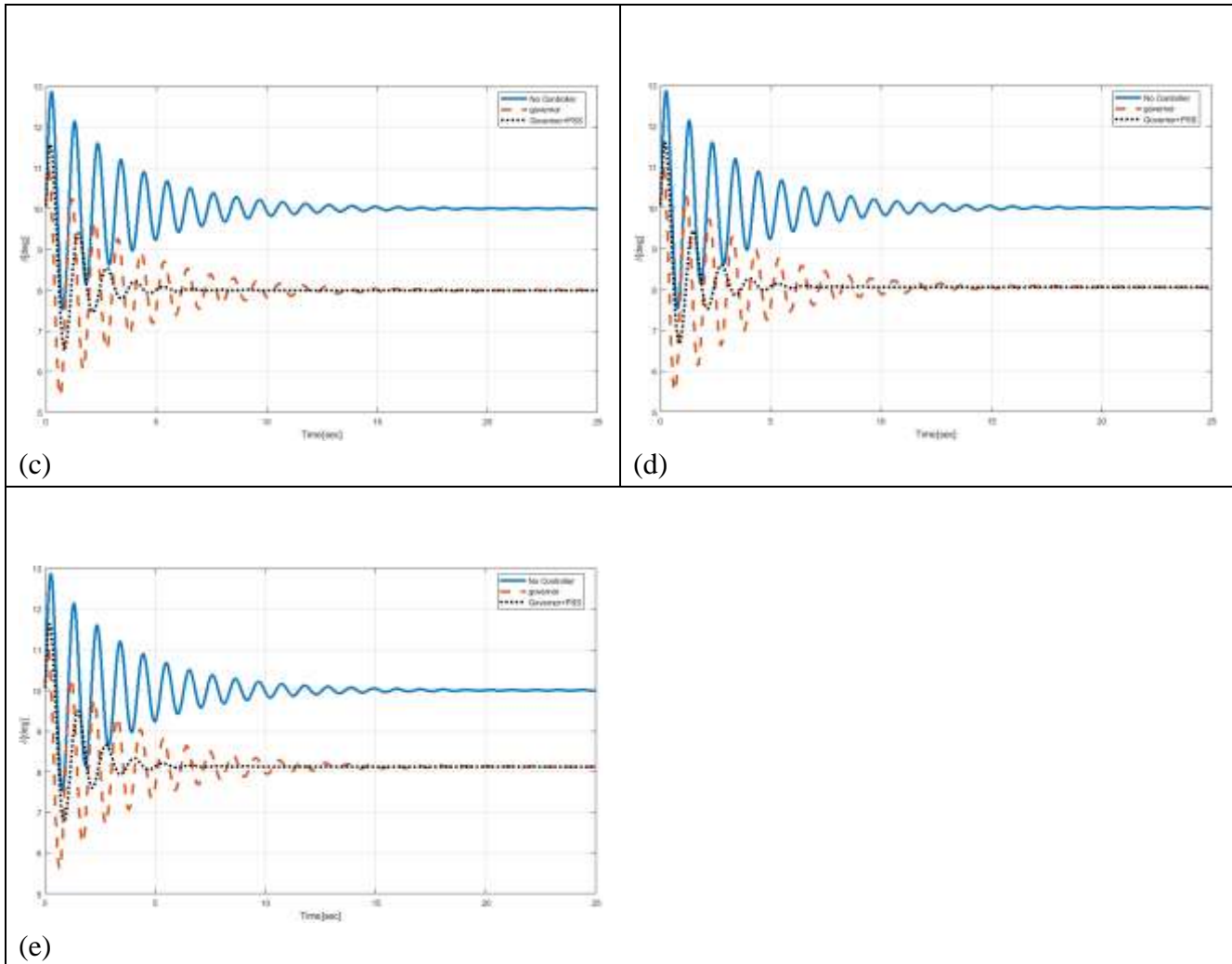


Figure 4: Rotor angle responses against time for the Kanji Generating Station

(a) 0% load: (b) 25% load. (c) 50% load and (d) 75% load (e) 100% load

Rotor angle oscillation waveforms for selected generators corroborate the eigenvalue results (Figures 1, 2, 3, and 4). Only the governor + PSS achieved damping times below the acceptable 25 s benchmark stipulated in industry standards (Jokojeje, 2024).

6. Discussion

The findings confirm that supplementary PSS control is indispensable for rotor angle stability in developing grids with weak interconnections. Governor-only schemes provide limited damping and are insufficient under stressed loading conditions (Falguni & Vijay, 2018).

Even with PSS, generators such as Delta show marginal stability, indicating the need for further controller refinement or network upgrades such as strengthening weak tie-lines or installing additional damping resources (Ma et al., 2020).

Our approach demonstrates the utility of model-based eigenvalue analysis as a predictive stability assessment and control design tool for system operators (Ghandhari, 2019).

7. Conclusion

This comprehensive eigenvalue-based study provides the following evidence:

- i. The rotor angle stability without control is precarious for Nigeria's 330 kV system

- ii. Governor + Power System Stabilizer control achieves satisfactory damping and settling times.
- iii. Some nodes require additional focus to ensure robustness across all operating scenarios.

This study provides a replicable framework for similar power systems in sub-Saharan Africa and other emerging grid contexts.

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Biography:



Dr. Rufus Akinnusimi Jokojeje holds a Ph.D. in Electrical Power Systems and Machines (2024) from the Federal University of Agriculture, Abeokuta. With over 15 years of academic experience at Moshood Abiola Polytechnic, Abeokuta, he has served as Sub Dean and Acting Head of the Electrical/Electronic Engineering Department. His research focuses on power system stability, compensator applications, and electrical network reliability. He is a registered engineer with COREN and a member of the Nigeria Society of Engineers. Dr. Jokojeje combines academic teaching, research, and professional engineering practice to advance

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Dr. Olujide Adeyinka Adenekan is an academic and engineer whose research spans power electronics and machine translation systems. He earned his PhD in Electrical & Electronics Engineering from Swansea University, where he contributed to advancements in low-voltage Power MOSFET modelling.

Before academia, he served at Moshood Abiola Polytechnic, Abeokuta, focusing on analogue/digital circuit design. Dr. Adenekan has co-authored papers on a rule-based English-to-Yorùbá translation system and obstacle-avoidance robotics, reflecting his interdisciplinary expertise. His innovative work blends engineering precision with linguistic computing, making him a notable figure in engineering research.

Temitope Grace Akinleye is an Electrical and Electronics Engineer whose research focuses on the application of Geographic Information Systems (GIS) in rural electrification and sustainable energy planning. She earned a B.Eng. in Electrical and Electronics Engineering from Ekiti State University and an M.Eng. with distinction from the Federal University of Agriculture, Abeokuta, where her dissertation applied GIS methodologies to improve rural energy access. She lectures at Moshood Abiola Polytechnic, teaching instrumentation, electrical machines, and building design, and serves as Technical Secretary of the NIEEE Abeokuta Chapter. Temitope aspires to pursue a PhD in GIS and renewable energy systems, emphasizing smart grids and sustainability.

Sahed Ademola Shittu is a First-Class graduate of Electrical and Electronics Engineering from the University of Ilorin, with research and project experience in power systems, renewable energy, and inverter design. He has contributed to teaching and applied research in power engineering while also building scalable software solutions for government and healthcare platforms. His expertise spans circuit analysis, PV system sizing, and control algorithms, complemented by backend engineering skills in microservices, cloud infrastructure, and database optimization.