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DEVELOPMENT OF A UKF-SLAM SCHEME FOR AUTONOMOUS NAVIGATION OF UNMANNED UNDERWATER VEHICLES

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Abstract: This paper presents a novel approach to the autonomous navigation of unmanned underwater vehicles (UUVs) using a simultaneous localization and mapping (SLAM) scheme based on an unscented Kalman filter (UKF). The proposed scheme employs a range sonar sensor to collect data and estimate the position of the UUV and surrounding objects in highly nonlinear motion scenarios. The UKF-SLAM scheme was validated through simulations and experiments, including two and three degrees of freedom motion conditions, in a towing tank environment with a real UUV. The results show that the proposed algorithm can accurately estimate the position of the UUV and surrounding objects and perform well under various conditions.

Keywords: simultaneous localization and mapping, UKF, unmanned underwater vehicles, navigation, range sonar, nonlinear motion, simulations, experiment.

Introduction

Autonomous navigation of unmanned underwater vehicles (UUVs) is becoming increasingly important in oceanographic research, underwater exploration, and military applications. Traditional navigation methods, such as inertial navigation systems and global positioning systems, are not always reliable in underwater environments due to signal attenuation and multipath effects (Chyba, 2009); the use of these vehicles has also extended to the inspection of ship hulls (Walter et al., 2008) and underwater man-made structures (Kondo et al., 2006; Ribas et al., 2008) due to the UUV ability of autonomous navigation. Since GPS signal is not accessible underwater, the position of the UUV has usually been estimated via dead reckoning using an inertia measurement unit (IMU) and a kinematics model for vehicle motion. However, the method based dead reckoning is disadvantageous in that the navigation error becomes unbounded as the navigation time elapses due to the drift produced when integrating the IMU's output; thus, it is necessary to provide some additional measures to prevent further error accumulation so that the ground fixed relative positioning information can be obtained (Lee et al., 2005). One of the alternative approaches that has been suggested to overcome the demerits of dead reckoning is a method that uses measured environmental data from the area through which the vehicle is passing; for example, it is possible to use the range and angle of identified objects that exist in the operating area as sources of information for a ground fixed relative position (Smith et al., 1997). Simultaneous localization and mapping (SLAM) is an alternative navigation method that measures the environment in which the vehicle is navigating and provides the relative position of the UUV. SLAM algorithms have been widely used in



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terrestrial and aerial robotics, but their application to UUVs is still limited. The highly nonlinear motion of UUVs in underwater environments makes the SLAM problem more challenging. (Smith et al., 1990). Since stochastic mapping, which is the basis of SLAM, is an estimation process, all estimation methods can be implemented into SLAM. Although any estimation method can be used in SLAM, many studies have applied an extended Kalman filter (EKF) to SLAM for indoor ground vehicles (Tardos et al., 2002), outdoor ground vehicles (Dissanyake et al., 2001), and underwater vehicles (Smith et al., 1997; Carpenter, 1998; Newman, 1999; Leonard and Feder, 2001; Hwang and Seong, 2005; Folkesson et al., 2008; Ribas et al., 2008) because EKF is considered optimal when assumptions that the system is locally liner and the probability density function (PDF) is a Gaussian distribution are satisfied (Welch and Bishop, 2006). However, if a nonlinear system cannot satisfy the locally linear assumption, the result estimated by the EKF for the nonlinear system might yield a divergence; thus, it is not appropriate to apply EKF to SLAM for highly nonlinear systems. One of the alternatives to a SLAM based EKF is a SLAM with an unscented Kalman filter (UKF). A UKF (Julier and Uhlmann, 1997, 2004) not only estimates the mean and covariance of the nonlinear system, as EKF does, but also obtains the transformed PDF from the weighted sum of the evaluation of the nonlinear system equation at samples of prior PDF, called sigma points, instead of the Jacobian matrix of the nonlinear system equation.

Up to now, since the UKF can produce a consistent unbiased estimate even when the system is highly nonlinear, recent studies have attempted to implement UKF into SLAM for unmanned land vehicles (Lee *et al.*, 2006; Andrade-Cetto *et al.*, 2005) and unmanned aerial vehicles (Langelaan and Rock, 2005), and to verify the SLAM with the UKF through simulations and experiments. However, although the motion of UUVs is usually nonlinear, there are few studies on SLAM with UKF for UUVs. Thus, a SLAM based on the UKF for UUVs will be proposed in this work and will be verified through simulations in tank conditions.

This paper proceeds as follows. Section 2 describes the SLAM based on the UKF that is suitable for UUVs with a range sonar system. Section 3 presents the results of the simulations in tank conditions, conducted to verify the proposed SLAM method. Section 4 concludes the paper.

Simultaneous localization and mapping formulation with unscented Kalman filter

A. State model

Consider the SLAM problem in which a UUV is navigating in the ocean with multiple objects located adjacent to where the vehicle is operating. If the UUV is assumed to move in three degrees of freedom, the state of the vehicle can be written as:

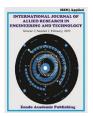
$$\boldsymbol{x}_{\boldsymbol{v}} = \begin{bmatrix} x_{\boldsymbol{v}} & y_{\boldsymbol{v}} & \psi_{\boldsymbol{v}} & \theta_{\boldsymbol{v}} & V_{\boldsymbol{v}} \end{bmatrix}^T, \tag{1}$$

where denote the position in a three dimesional space; denote the heading and pitch angle, respectively, and denotes the total velocity. As the motion of the UUV is assumed to be divided into two planes, vertical and horizontal planes in this work, only surge, yaw, and pitch are considered in (1). The state of M objects acquired by the sensor on the UUV is given by their positions:

$$\mathbf{x_o} = [x_{o1} \quad y_{o1} \quad z_{o1} \quad \cdots \quad x_{oM} \quad y_{oM} \quad z_{oM}]^T.$$
 (2)

The objects 'positions are assumed to be fixed and unassociated with each other.

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Since the system state vector in SLAM is defined as the combination of a vehicle state and an object's state, the system state vector in this work is as follows:

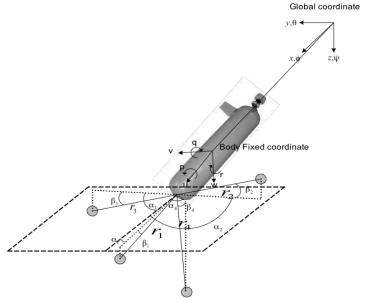
$$\boldsymbol{x} = [\boldsymbol{x}_{v}^{T} \quad \boldsymbol{x}_{o}^{T}]^{T}. \tag{3}$$

In this work, the vehicle motion is modeled with a second kinematics model, which is characterized by a constant velocity with white noise acceleration (Bar-Shalom and Forman, 1988). The discrete time state equation at time k and k+1 with sampling time is described as:

$$x_{o}(k+1) = x_{o}(k), \qquad (4)$$
 where
$$x_{v}(k+1) = x_{v}(k) + V_{v}cos(\theta_{v}+r)cos(\theta_{v}+q) \triangleq T + q_{x}(k)y_{v}(k+1) \qquad q_{x}, q_{y}, q_{z}, q_{\psi}, q_{\theta}, q_{v}$$
 are
$$y_{v}(k) + V_{v}sin(\theta_{v}+r)cos(\theta_{v}+q) \triangleq T + q_{y}(k)z_{v}(k+1) \qquad \text{the process noises}.$$
 B.
$$z_{v}(k) + V_{v}sin(\theta_{v}+q) \triangleq T + q_{z}(k) \qquad \text{Measurement model}$$
 In this
$$\psi_{v}(k+1) = \psi_{v}(k) + r + q_{\psi}(k) \qquad \text{work, a range sonar system with four}$$

$$\theta_{v}(k+1) = \theta_{v}(k) + q + q_{\theta}(k) \qquad \text{channels is considered for the device to}$$
 obtain
$$V_{v}(k+1) = V_{v}(k) + q_{v}(k) \qquad \text{the spatial information of objects in the}$$
 environment in which the UUV is navigating. Although the range sonar system can provide only the range, the measurements for the SLAM in this work are the distance and angles between the UUV and the objects, because it is able to obtain the angle information from the relationship of the UUV position and the fixed position of the range sonar sensor. Figure 1 shows the concepts of the measurement component, distance, and angles.

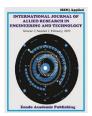
Figure 1 Coordinate system and the measurement of range and angle.



The measurement model for the four channel range sonar system is described as follows:

$$\mathbf{z}(k+1) = \begin{bmatrix} \mathbf{z}_{1}(k+1) \\ \mathbf{z}_{2}(k+1) \\ \mathbf{z}_{3}(k+1) \\ \mathbf{z}_{4}(k+1) \end{bmatrix} + \begin{bmatrix} \mathbf{w}_{1}(k+1) \\ \mathbf{w}_{2}(k+1) \\ \mathbf{w}_{3}(k+1) \\ \mathbf{w}_{4}(k+1) \end{bmatrix}.$$
(5)

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The subscript in (5) denotes the corresponding direction in the range sonar system. The orders of direction for the range sonar are front, starboard, port, and downward. The measurement model for single objects detected by the i^{th} range sonar at time k+1 is described as follows:

$$\mathbf{z}_{i} = \begin{bmatrix} r_{i}(k+1) \\ \alpha_{i}(k+1) \\ \beta_{i}(k+1) \end{bmatrix} + \mathbf{w}(k+1)$$
(6)

In (6), $r_i(k+1)$ is the distance, $\alpha_i(k+1)$ is the angle rotated from the z axis, $\beta_i(k+1)$ is the angle rotated from the y axis, and w(k+1) represents the measurement noises of the sonar, which are assumed to have a zero mean white Gaussian noise with the covariance dependent on the sonar specifications.

C. SLAM formulation

The first step of the proposed scheme is the calculation of sigma points, which is the essential component of UKF. The prediction at time k+1 for the state system vector described in (3) is performed using the calculated sigma points through the weight sum method of the UKF. The prediction of the measurement is performed in the same manner as the prediction of the system state vector. Following the prediction, the measurement is made. After the measurement step, the detection of new objects is decided by 'data association', which determines whether the measurements obtained by the sonar are to be labeled as new objects or as existing objects already registered on the map. Data association is one of the most important SLAM factors for practical SLAM implementation because one incorrect data association can introduce divergence into the map estimate. Even though data association is an important problem in SLAM, in this work, the nearest neighborhood standard filter (NNSF) (Bar-Shalom and Forman, 1988) is used as the data association method instead of developing a new data association algorithm because the purpose of this paper is the implementation the SLAM based UKF into small UUVs. Measurements that do not correspond to existing objects through data association based on NNSF are considered as new objects and are registered. Whenever a new object is detected, it is added to the existing system state vector and covariance matrix using the stochastic map.

As the operation time of the UUV increases, the sizes of the system state vector and its covariance matrix increase in identical proportions as a number of newly detected objects are added; the computational burden grows as $O(M^2)$, where M is the size of the system state vector. This computational burden can become very large as the operation time increases. Therefore, it is necessary to adopt a method to reduce the computational burden. Previous studies have provided several alternatives that can reduce the computational burden (Newman 1999; Williams *et al.* 2002). In this paper, the local submap method (Kim, 2004) is adopted. In the local submap method, the environment in which the UUV is navigating is divided into several local submaps, and the full covariance method is used to estimate the positions of the UUV and objects in a single submap.



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Simulations in towing tank condition

A. Conditions

In order to verify the proposed method, several simulations were performed. It was assumed that the UUV is navigating in a water tank with a length of 120 m, breadth of 8 m, and depth of 3.5 m, while detecting the walls and floor of the tank and extra objects lying within the tank. Four sets of simulation conditions were chosen as shown in Table 1. Sets 1 and 2 are cases for two degrees of freedom motion in which the yaw varies with small and large angular motions; Sets 3 and 4 are those for three degrees of freedom motion in which the yaw and pitch vary. In Set 3, the heading and pitch change sequentially, whereas in Set 4 both angles change simultaneously. The extra objects are assumed to be located regularly in the tank. Three channels (forward, left, and right) of the range sonar were used for the 2 degrees of freedom motion and an extra channel (downward) was used for the 3 degrees of freedom motion. A comparison between the proposed method and EKF based SLAM was performed to verify the function for UKFSLAM, which is proposed in this paper under nonlinear conditions. The basic parameters of the EKF are shown in Table 2. The standard deviation of acceleration, heading, and pitch were decided considering the specification of the SNUUV I under development.

Table 1. Simulation conditions

No	UUV	No. of	No. of extra	UUV angle variation
	Velocity	channels	objects	
1	0.1, 0.3, 0.5 m/s	3 ch.	4-8 objects	-5 ~ 5 °
2	0.1, 0.3, 0.5 m/s	3 ch.	4-8 objects	-10 ~ 10 °
3	0.1, 0.3 m/s	4 ch.	0	-10 ~ 10 ° (respectively change of heading and pitch)
4	0.1, 0.3 m/s	4 ch.	0	-10 ~ 10 ° (concurrent change of heading and pitch)

Table 2. Basic parameters of EKF

F		
Parameters	Value	
Variance of vehicle acceleration	0.01 m/s^2	
Variance of vehicle angles	1°	
Variance of object position	1 m	
Variance of range	1 m	
Variance of angles	5°	

B. Simulation results

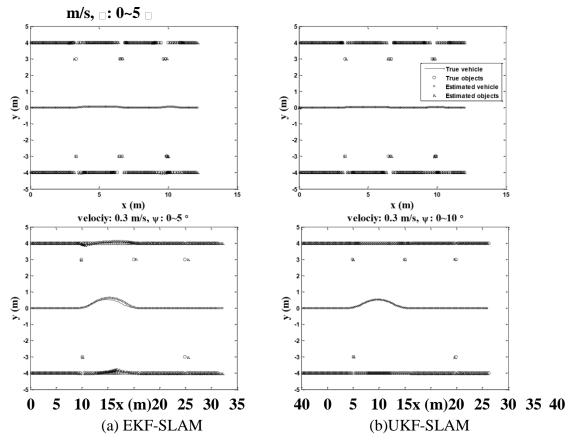
Figure 2 shows several representative results for the 2 degrees of freedom motion simulation. The origin of the 2 dimensional map is the UUV starting point. The true positions of the UUV and objects (continuous



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walls and five isolated targets) are designated by a solid line (-) and circles (\circ), respectively, while the estimated positions of the vehicle and objects are shown using crosses (+) and x marks (x), respectively. The left and right column figures are the EKF-SLAM and UKF-SLAM results, respectively. From Fig. 2, when the nonlinearity of the UUV motion is weak (when the UUV velocity is slow and yaw variance is small), both methods show little error in the mapping and localization results; however, when the UUV moves quickly or the yaw varies with large angles, the UKF-SLAM provides more accurate mapping and localization results than the EKLSLAM. This verifies that the method proposed in this paper is suitable for the nonlinear motions of UUVs. Figure 3 shows the UUV location estimation errors, i.e. the difference between the solid line and the crosses in Fig. 2. The solid line is the error from the UKFSLAM and the dashed line is that of the EKF-SLAM. The EKF error increases once the heading angle begins to change at time 30 sec; that is, the nonlinearity occurred in the state transition as predicted in the theory.

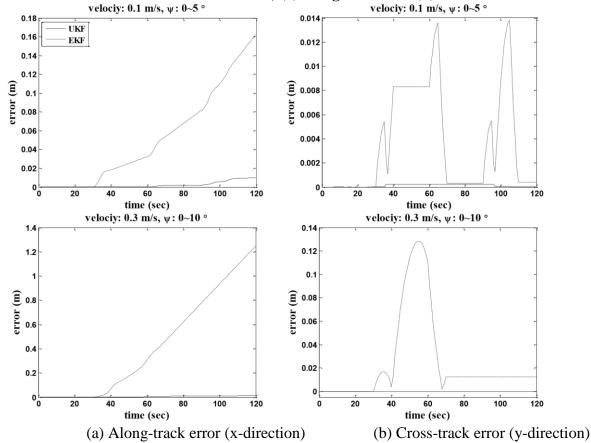
Figure 2. Simulation results of (a) EKF-SLAM and (b) UKF-SLAM for 2D motion depending on the heading angle changes and vehicle velocity. velociy: 0.1 m/s, \Box : $0\sim5$ \Box velociy: 0.1





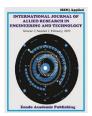
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Figure 3. . Comparison of estimation errors of the UUV location using EKF-SLAM and UKF-SLAM in the 2D motion simulation; (a) along-track and (b) cross-track directions.



The root mean square error (RMSE) of the EKF-SLAM is more than twice that of the proposed method. From Fig. 3, the RMSEs of the proposed method in the *x* and *y* directions are 0.233 m and 0.004 m in the upper section of Fig. 3, and 0.191 m and 0.004 m in the lower section of Fig. 3; however, the RMSEs of EKF-SLAM are 0.436 m and 0.014 m in the upper section of Fig. 3, and 0.428 m and 0.025 m in the lower section of Fig. 3.

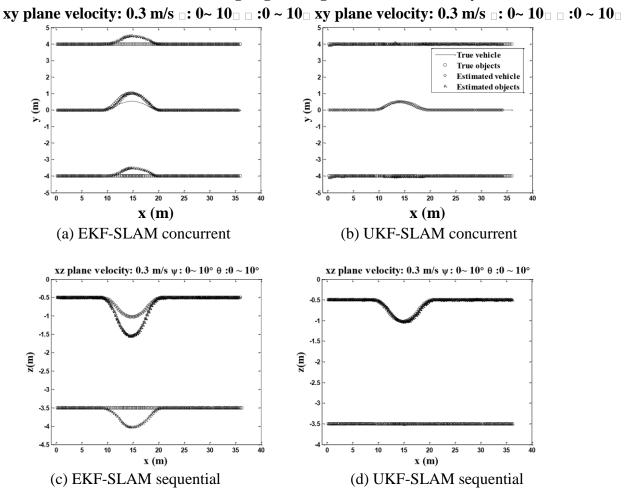
The simulation results of the three degrees of freedom motion are depicted in Fig. 4. The difference in performances of both methods is seen in the 3D motion simulation, where the nonlinearity is generally stronger. Figure 4(a) and 4(b) show the EKF-SLAM and UKFSLAM simulation results for the Set 3 motion from Table 1 and Figs. 4(c) and 4(d) present the both method simulation results for the Set 4 motion from Table 1. The upper section of the Figs. 4(a), 4(b), 4(c) and 4(d) is the horizontal plane (x-y) results and the lower is the vertical plane (x-z) result. The same symbols in Fig. 2 are used to describe the true positions of the UUV and objects, the estimated positions of the vehicle and objects in Fig. 5. Figs. 4(a) and 4(b) prove the UKF-SLAM can produce the more accurate mapping and localization results than the EKF-SLAM when the heading and pitch vary simultaneously. The sequential motion results shown in Figs. 4(c) and 4(d) also show prove the UKF-SLAM is more suitable for nonlinear case than EKF-SLAM. Fig. 5 shows that the UKF-



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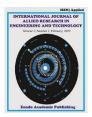
SLAM provides more accurate mapping and localization results than the EKL-SLAM when the UUV motion is the three degrees of freedom motion. The simulation results clearly prove that the proposed method has better performance than that of EKF-SLAM when nonlinearity is strong.

Figure 4. Simulation results of (a) EKF-SLAM and (b) UKF- SLAM for 3D motion depending on the heading angle changes and vehicle velocity.



Conclusion

This paper presents a SLAM method that relies solely on range sonar data, which is suitable for a small UUV with a limited payload and computing power. The method uses an unscented Kalman filter to estimate the system state vector with multiple objects using a nonlinear system equation instead of a linearized equation when the UUV navigates in a three dimensional space. It also adopts the nearest neighborhood standard filter for data association and the local submap method to reduce the computational power. The proposed method was tested through computer simulations under various conditions of varying the vehicle velocity, ranges, heading angles, and pitch angles. The simulation results showed that the proposed method produces better localization of the vehicle state and objects mapping in both two dimensional and three dimensional motions than that of the EKF-SLAM. Future works will investigate the



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feasibility of integrating SLAM into the total navigation system of real UUVs where it will function as a module that provides information about the surrounding environment and to experiment with the UUV and the proposed SLAM method in a fully autonomous navigation mode in the towing tank.

References

- Andrade-Cetto, J., Vidal-Calleja, T., Sanfeliu, A., 2005. Unscented transformation of vehicle state in SLAM. Proc. IEEE ICRA, pp. 323-328.
- Bar-Shalom Y., Forman T., 1988. Tracking and Data Association, Academic Press.
- Carpenter, R.N., 1998. Concurrent mapping and localization with FLS. Proc. AUV98, pp.
- 133 148.
- Chyba, M., 2009. Autonomous underwater vehicles. Ocean Engineering 36 (1), 1. Dissanyake, M., Newman, P., Clark, S., Durrant-Whyte, H., M. Csobra, 2001. A solution to simultaneous localization and map building(SLAM) problem. IEEE Trans. on Robotics and Automation, Vol.17, No.3, 229-241.
- Folkesson, J., Leederkerken, J., Williams, R., Patrikalakis, A., Leonard, J., 2008. A Feature based navigation system for and autonomous underwater robot. Springer Tracts in Advanced Robotics, Vol. 42, p 105-114.
- Hwang, A., Seong, W., 2005. Concurrent mapping and localization simulation and experiment using range sonar data. UUST05.
- Julier, S.J., Uhlmann, J.K., 1997. A new extension of the Kalman filter to nonlinear systems. Proc. of AeroSense: The 11th Int. Sym. on Aerospace/Defense Sensing, Simulation and Controls, SPIE.
- Julier, S.J., Uhlmann, J.K., 2004. Unscented filtering and nonlinear estimation. Proc. of IEEE Vol. 92, No. 3, pp. 401-422.
- Jun, B., Park, J., Lee, F. Lee, P., Lee, C., Kim, K., Lim Y., Oh, J., 2009. Development of the AUV 'ISiMI 'and a free running test in an Ocean Engineering Basin, Ocean Engineering Vol. 36, No. 1, pp. 2–14.
- Kim, S.J., 2004. Efficient simultaneous localization and mapping algorithm using submap networks. PhD Thesis, MIT.
- Langelaan, J., Rock, S., 2005. Passive GPS-free navigation for small UAVs. IEEE Aerospace Conference, pp. 1-9.
- Kondo, H., Maki, T., Ura T., Sakamaki, T., 2006. AUV navigation based on multi-sensor fusion for breakwater observation. ISARC2006, pp. 72-77.



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- Lee, C., Lee, P., Hong, S., Kim, S., Seong, W., 2005. Underwater navigation system based on inertial sensor and Doppler velocity log using indirect feedback Kalman filter. Int. J. Offshore and Polar Eng., Vol. 15, No.2, pp. 88-95.
- Lee, S., Lee, S., Kim, D., 2006. Recursive unscented Kalman filtering based SLAM using a large number of noisy observations. Int. J. of Control, Automation, and Systems, Vol. 4, No. 6, pp. 736-747.
- Leonard, J., Feder, H., 2001. Decoupled stochastic mapping. IEEE J. Oceanic. Eng. Vol. 26, No. 4, pp. 561-571.
- Maki, T. Kondo, H. Ura, T. Sakamaki, T., 2006. Photo mosaicing of Tagiri shallow vent area by the AUV "Tri-Dog 1" using a SLAM based navigation scheme, OCEANS 2006, Martinez-Cantin, R., Castellanos, J.A., 2005. Unscented SLAM for large-scale outdoor environments. IEEE/RSJ Int. Conf. on Intelligent Robots and Systems, pp. 328-333.
- Montemerlo, M., Thrun, S., Koller, D., Wegbreit, B., 2002. FastSLAM: A factored solution to the simultaneous localization and mapping problem. In: AAAI National Conference on Artificial Intelligence, pp. 593-598.
- Newman, P.M., 1999. On the structure and solution for simultaneous localisation and mapping problem. PhD Thesis, The University of Sydney.
- Ribas, D., Ridao, P., Tardos, J.D., Neira, J., 2008. Underwater SLAM in man-made structured environments, Journal of Field Robotics, Vol. 25, No. 11-12, pp. 898-921.
- Smith, C., Leonard, J., Bennett, A., Shaw, C., 1997. Feature-based concurrent mapping and localization for autonomous underwater vehicles. Proc. MTS/IEEE Int. Conf. Oceans'97, pp. 896-901.
- Smith, R., Self, M., Cheeseman, P., 1990. Estimating uncertain spatial relationship. In: Autonomous Robot Vehicles, Spring-Verlag. I. Cox, G. Wilfong (Eds.).
- Tardos, J.D., Neira, J., Newman, P., Leonard J., 2002. Robust mapping and localization in indoor environments using sonar data. The International Journal of Robotics Research, Vol. 21, No. 4, pp. 311 -330.
- Walter, M., Hover, F., Leonard, J., 2008. SLAM for ship hull inspection using exactly sparse extended information filters, Proc. IEEE Int. Conf. Robotics and Automation, pp. 1463-1470.
- Welch, G., Bishop, G., 2006. An introduction to the Kalman filter, TR 95-041. Dept. of Computer Science, University of North Carolina.
- Williams, S.B., Dissanayake, G., Durrant-Whyte, H., 2002. An efficient approach to the simultaneous localization and mapping problem. Proc. IEEE Int. Conf. Robotics and Automation, pp. 406-411.