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INVESTIGATING THE NIGERIAN NAIRA PER US-DOLLAR EXCHANGE RATE (NGNUSD) USING ARMA-GARCH MODELS

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Abstract

Exchange rate volatility plays a crucial role in economic stability, trade balance, and investment decisions. The Nigerian Naira to US Dollar (NGN/USD) exchange rate has experienced significant fluctuations due to macroeconomic factors, global oil price movements, and monetary policies. This study investigates the statistical properties, dynamics, and volatility clustering of the NGN/USD exchange rate using Autoregressive Moving Average (ARMA) and Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models. This study uses historical daily exchange rate data from July 1995 to June 2024, sourced from the Central Bank of Nigeria (CBN) and other financial institutions. The methodology involves time-series econometric techniques, including stationarity tests, ARMA modeling for mean dynamics, and GARCH modeling for volatility estimation. Model selection criteria, such as the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) were used to identify the best-fitting model. The results indicate that the NGN/USD exchange rate exhibits volatility clustering, where periods of high volatility are followed by further fluctuations. The ARMA (1,1)-GARCH(1,1) model was found to be the most suitable model for capturing both the mean and variance structure of the exchange rate. The persistence of volatility suggests that external shocks, speculative trading, and macroeconomic conditions significantly impact Naira stability. The study concludes that exchange rate volatility in Nigeria poses a key economic challenge that requires strategic policy interventions. Based on these findings, adopt robust foreign exchange management strategies, financial institutions should integrateadvanced risk assessment models, and future research should explore additional macroeconomic factors affecting exchange rate dynamics.

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1. INTRODUCTION

The exchange rate is a critical economic indicator that influences macroeconomic stability, trade balances, inflation rates, and overall economic growth. The Nigerian Naira per US-Dollar exchange rate (NGN/USD) has historically exhibited significant volatility due to various macroeconomic and external shocks, including fluctuations in global oil prices, monetary policy adjustments, and foreign investment flows (Aliyu, 2022; Ekanem & Uchenna, 2023).

Modeling exchange rate movements is essential for policymakers, investors, and economic analysts to develop informed risk management and policy formulation strategies. Traditional time-series models, such as the Autoregressive Moving Average (ARMA) model, are widely used to model financial time-series data (Box & Jenkins, 1976). However, financial time-series data often exhibit volatility clustering, which suggests that large price movements tend to be followed by large movements of either sign (Engle, 1982; Bollerslev, 1986). To account for such characteristics, Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models are used to model and forecast volatility in exchange rate series.

Given the importance of exchange rate stability for economic planning and financial decision-making, this study investigates the behavior of NGN/USD using ARMA-GARCH models. By integrating ARMA to capture linear dependence and GARCH to model volatility clustering, this study seeks to provide an empirical basis for understanding the dynamics of the Nigerian exchange rate.

LITERATURE REVIEW

2 Introduction

This chapter reviews relevant literature on exchange rate dynamics, volatility modeling, and the application of ARMA-GARCH models to financial time series analysis. This review encompasses theoretical foundations, empirical studies, and conceptual frameworks that provide insight into exchange rate behavior in Nigeria.

3 Conceptual Framework

The conceptual framework for this study is based on the interaction between exchange rate dynamics and volatility modeling using the ARMA-GARCH approach. Exchange rate fluctuations are influenced by macroeconomic fundamentals, speculative activities, and external shocks. Traditional models, such as purchasing power parity (PPP) and Interest Rate Parity (IRP), explain exchange rate movements based on economic fundamentals (Krugman & Obstfeld, 2003). However, financial time series data often exhibit volatility clustering, which necessitates the application of models that account for heteroskedasticity.

In the Nigerian context, exchange rate volatility is driven by factors such as inflation rates, interest rate differentials, trade imbalances, foreign direct investment, and external economic shocks (Adegbite & Hassan, 2023). The fluctuations in the Naira-to-US-Dollar exchange rate (NGNUSDER) have significant implications for inflationary trends, investment decisions, and economic stability. Understanding the pattern of exchange rate volatility is crucial for policymakers, investors, and financial analysts.

The ARMA model is used to capture linear dependencies in exchange rate time series, whereas the GARCH model accounts for time-varying volatility (Engle, 1982; Bollerslev, 1986). The integration of these models provides a robust framework for analyzing exchange rate behavior, especially in volatile economies, such as Nigeria.

Macroeconomic indicators play a crucial role in determining exchange rate fluctuations by shaping investor sentiment, influencing capital flows, and reflecting the overall economic health of a nation. A country’s gross domestic product (GDP) growth rate is a key indicator of economic performance and competitiveness. Higher

GDP growth attracts foreign direct investment (FDI) and portfolio investment, increasing demand for the domestic currency and leading to its appreciation (Obstfeld & Rogoff, 1996). Conversely, sluggish or negative GDP growth discourages investment, causing capital flight and weakening the currency (Frankel & Rose, 1995). Inflation affects exchange rates by influencing purchasing power. A country experiencing high inflation relative to their trading partners tend to see their currency depreciate as the cost of goods and services rises, reducing their competitiveness in global markets. Conversely, low and stable inflation strengthens a currency by preserving purchasing power and attracting investment (Dornbusch, 1976). Central banks use monetary policy to influence exchange rates through interest rate adjustments. Higher interest rates generally attract foreign capital, increasing demand for the currency and leading to appreciation. Conversely, lower interest rates can result in capital outflows, reducing demand for currency and causing depreciation (Mundell, 1963). The interest rate differential between countries also plays a role in exchange rate movements, as investors seek higher returns on assets (Taylor, 1995). The trade balance, which measures the difference between exports and imports, directly affects demand for currency. A country with a trade surplus earns more foreign currency from exports, increasing demand for its domestic currency and strengthening its exchange rate. Conversely, trade deficits lead to higher demand for foreign currencies, causing depreciation (Krugman & Obstfeld, 2003).

Speculative trading, market expectations, and capital flows play crucial roles in short-term exchange rate fluctuations by influencing supply and demand dynamics in foreign exchange markets. These factors often lead to volatility, as traders and investors react to economic indicators, geopolitical events, and market sentiment (Ekanem & Uchenna, 2023).

Exchange rates exhibit both predictable and unpredictable fluctuations due to macroeconomic fundamentals, market expectations, and speculative activities. Time-series volatility modeling plays a crucial role in capturing these dynamics, with the **Autoregressive Moving Average - Generalized Autoregressive Conditional Heteroskedasticity (ARMA-GARCH)** model being a widely used framework. This model accounts for both the mean and variance structures of exchange rate movements, allowing for better forecasting and risk assessment (Bollerslev, 1986)

4 Theoretical Framework

Several theories have been proposed to explain exchange rate movements and volatility. Some of the most relevant theories are as follows

4.1 Purchasing Power Parity (PPP) Theory

The Purchasing Power Parity (PPP) theory asserts that the exchange rate between two currencies is primarily determined by the relative price levels of the two countries involved (Cassel, 1918). According to this theory, in the long run, exchange rates should adjust to ensure that identical goods and services cost the same in different countries when measured in a common currency. This principle is based on the Law of One Price, which states that in the absence of transportation costs, trade barriers, and market frictions, arbitrage drives price convergence across countries.

However, empirical evidence in the short run suggests that exchange rates frequently deviate from PPP due to several factors. Inflation differentials between countries can cause temporary distortions because a country experiencing higher inflation may see its currency depreciate relative to a country with lower inflation. Interest rate changes also influence exchange rates, as higher interest rates attract foreign capital inflows, leading to currency appreciation, whereas lower interest rates may trigger capital outflows and depreciation. Additionally, external shocks, such as geopolitical tensions, financial crises, or sudden changes in trade policies, can cause exchange rates to diverge from their PPP-predicted values (Taylor & Taylor, 2004).

Despite short-term deviations, PPP remains a useful benchmark for assessing long-term exchange rate trends. Over extended periods, exchange rates tend to move toward their fundamental equilibrium levels, driven by adjustments in inflation, trade balances, and relative price levels. However, the speed and extent of this adjustment depend on factors such as market efficiency, central bank policies and structural economic differences between countries.

4.2 Interest Rate Parity (IRP) Theory

The **Interest Rate Parity (IRP) theory** posits that the exchange rate between two currencies is primarily determined by the difference in interest rates between the two countries (Fama, 1984). This principle is grounded in the idea that investors seek to maximize returns by allocating capital to markets that offer higher interest rates. Consequently, interest rate differentials exist, they trigger capital flows, influence exchange rate movements and ensure equilibrium in the foreign exchange market. IRP is classified into two main forms:

- **Covered Interest Rate Parity (CIRP):** This parity occurs when the relationship between interest rates and exchange rates is enforced through forward contracts, eliminating the possibility of arbitrage. According to the CIRP, the forward exchange rate should adjust to offset the interest rate differential between the two countries, ensuring no risk-free profit opportunities (Frankel, 1992).
- **Uncovered Interest Rate Parity (UIRP):** Unlike CIRP, UIRP assumes that no forward contracts are used, and exchange rates adjust based on market expectations. This suggests that a country with a higher interest rate will experience expected depreciation in its currency to offset the higher returns offered to investors (Meese & Rogoff, 1988).

When a country has a higher **nominal interest rate** than another, investors are incentivized to move capital into that country to earn better returns. This inflow increases demand for higher- yielding currency, leading to short-term appreciation. However, in the long run, exchange rates adjust to reflect the interest rate differential, ensuring that no arbitrage opportunities exist for risk-free profit (Dornbusch, 1976).

For example, if U.S. interest rates are higher than those in Japan, investors will convert the Japanese yen to U.S. dollars to invest in U.S. assets. This increased demand for the dollar causes the U.S. currency to appreciate against the yen. However, the exchange rate should eventually adjust to equalize expected returns between the two markets.

Although IRP provides a theoretical foundation for exchange rate determination, real-world deviations occur because of several factors:

- **Market Imperfections:** Transaction costs, capital controls, and differing risk perceptions can prevent full arbitrage, leading to deviations from the IRP (Froot & Thaler, 1990).
- **Exchange Rate Expectations:** Speculations and uncertainty about future exchange rate movements can cause short-term fluctuations that do not align with IRP predictions (Bekaert & Hodrick, 1993).
- **Monetary Policy and Central Bank Interventions:** Central banks often influence interest and exchange rates through monetary policy tools, such as open market operations, which may disrupt the IRP equilibrium (Taylor, 1995).

Empirical studies have found that IRP holds in many cases, particularly in well-developed financial markets, but short-term deviations are common because of speculative behavior and risk premiums. However, over the long term, exchange rates tend to align with interest rate differentials, reinforcing the fundamental principles of IRP.

4.3 Balance of Payments (BOP) Theory

The **Balance of Payments (BOP) theory** asserts that a country’s exchange rate is fundamentally influenced by its economic transactions with the rest of the world, including trade in goods and services, capital flows, and financial transfers (Krugman & Obstfeld, 2003). The BOP is a comprehensive record of all monetary exchanges between a country and its international trading partners, and its overall balance directly affects the demand and supply of the national currency in the foreign exchange market. The BOP consists of two main accounts that determine exchange rate fluctuations: This account records trade in goods and services, income from foreign investments, and unilateral transfers. A **current account surplus** (when exports exceed imports) leads to higher demand for the domestic currency, resulting in appreciation. A **current account deficit** (when imports exceed exports) increases the supply of domestic currency in the forex market, leading to depreciation.

This account captures capital flows, such as foreign direct investments (FDI), portfolio investments, and loans. Net **capital inflows** (when foreign investments exceed outflows) create demand for the local currency, leading to appreciation. **Net capital outflows** (when domestic investors move capital abroad) increase the supply of local currency in the forex market, causing depreciation.

Persistent trade deficits indicate that a country imports more than it exports, requiring continuous purchases of foreign currency finance the shortfall. This **excess supply of domestic currency** weakens its value, leading to depreciation. Conversely, a country with consistent trade surpluses earns more foreign exchange, which increases demand for its currency and drives appreciation.

For example, China's sustained trade surplus has historically contributed to the **appreciation of the yuan**, whereas the **U.S. trade deficit** has placed downward pressure on the dollar, necessitating capital inflows to maintain its value (Obstfeld & Rogoff, 1995). In addition to trade balances, capital flows play a crucial role in determining exchange rerates. henmultinational corporations invest in a country, they increase demand for its currency, leading to appreciation. High-interest rates or strong stock market performance attract foreign investors, increasing capital inflows and boosting the currency's value. Investors often shift capital in response to expected exchange rate movements, sometimes causing excessive volatility in currency markets (Dornbusch, 1976).

For instance, **emerging markets** with high interest rates and growth potential often experience appreciation in currency markets due to **strong foreign capital inflows**. Conversely, when investors lose confidence in a country's economy, they withdraw capital, leading to depreciation.

While BOP dynamics largely determine exchange rates, external shocks, such as financial crises, geopolitical instability, and sudden shifts in global trade patterns, can disrupt the equilibrium. Governments and central banks may intervene to stabilize currency markets through buying or selling foreign currencies to influence exchange rates. Changes in interest rates can attract or deter capital flows, indirectly affecting exchange rates. Tariffs and export restrictions can affect trade balances and influence currency movements.

For example, Japan's **interventions in the yen market** to control excessive appreciation have demonstrated that governments actively manage exchange rates (Frankel, 1992). Empirical studies have found that while BOP theory explains long-term exchange rate trends, short-term fluctuations are often driven by speculation and investor sentiment. However, **sustained trade imbalances or prolonged capital outflows** ultimately exert significant pressure on exchange rates, reinforcing the relevance of the BOP framework in currency valuation.

5. Exchange Rate Volatility and Its Determinants

Exchange rate volatility refers to fluctuations in the value of a currency relative to another currency during a specific period. These fluctuations can be influenced by several economic, political, and financial factors thatcontribute to the instability of exchange rates. Understanding these factors is essential for policymakers, businesses, and investors in managing risks associated with foreign exchange movements.

1. Macroeconomic Indicators

Macroeconomic fundamentals, such as inflation, interest, and gross domestic product (GDP) growth, significantly impact exchange rates. Higher inflation rates in a country tend to devalue its currency as purchasing power declines, leading to depreciation relative to other currencies (Mishkin, 2019). Similarly, interest rate differentials influence capital flows, as investors seek higher returns in economies with more attractive rates. For instance, when the Central Bank of Nigeria (CBN) raises interest rates, it may attract foreign investments, leading to an appreciation of the naira. GDP growth also plays a role, as stronger economic performance attracts foreign direct investments (FDIs), enhancing currency stability.

2. Monetary Policy

Decisions made by central banks, particularly the Central Bank of Nigeria (CBN), directly affect exchange rate stability. Changes in monetary policy, such as adjustments to the money supply, open market operations, and foreign exchange interventions, play a crucial role in managing exchange rate volatility (CBN, 2023). For example, when the CBN injects more liquidity into the economy, it can lead to currency depreciation. Conversely, measures like tightening monetary policy by increasing interest rates or selling foreign exchange reserves can stabilize or strengthen the currency. The effectiveness of these policies depends on broader economic conditions and external market forces.

3. Global Economic Shocks

Exchange rate volatility is often exacerbated by global economic shocks, which include fluctuations in commodity prices, financial crises, and geopolitical tensions (Adebayo & Akinwale, 2022). Nigeria, an oil-dependent economy, is particularly susceptible to fluctuations in global oil prices. A sharp decline in oil prices reduces foreign exchange earnings, leading to currency depreciation. Additionally, global financial crises can trigger capital flight from emerging markets, including Nigeria, resulting in increased volatility in exchange rates. Geopolitical events such as trade wars, international conflicts, and economic sanctions also contribute to unpredictable fluctuations in currency markets

4. Market Speculation

Investor sentiment and speculative activities in the foreign exchange market significantly influence short-term exchange rate movements. Traders and investors make currency decisions based on expectations of future price movements, which can lead to rapid fluctuations (Froot & Thaler, 1990). Speculators may drive currency appreciation or depreciation through large-scale foreign exchange buying or selling. For instance, rumors or news about potential policy changes, economic instability, or political uncertainty can prompt speculative attacks on the naira, worsening volatility.

6. ARMA-GARCH Models for Exchange Rate Analysis

Time series modeling of exchange rates requires capturing both mean and variance dynamics. Exchange rates exhibit volatility clustering, meaning periods of high volatility tend to be followed by high volatility and vice versa. To effectively model such behavior, the **Autoregressive Moving Average (ARMA)** model is used to describe the mean structure, whereas the **Generalized Autoregressive Conditional Heteroskedasticity (GARCH)** model captures time-varying volatility. The ARMA-GARCH framework is widely employed in financial time-series analysis due to its ability to model both the conditional mean and conditional variance of a time series.

6.1 Autoregressive Moving Average (ARMA) Model

The ARMA model, which was introduced by **Box and Jenkins (1976)**, is used to model stationary time-series data by incorporating two components:

- **Autoregressive (AR) Component**: This component captures the relationship between the current exchange rate value and its past values.
- **Moving Average (MA) Component**: This component models the dependence of the exchange rate on past forecast errors.

Mathematically, an **ARMA (p, q)** process for a time series X_t (e.g., exchange rate) is defined as follows:

$$X_t = c + \sum_{i=1}^{p} \emptyset_i X_{t-1} + \sum_{j=1}^{q} \theta_j \epsilon_{t-j} + \epsilon_t$$

Where:

- X_t is the exchange rate at time ttt, and
- c is a constant term.
- $\emptyset_i$ (for i=1,2,...,p) are the coefficients of the autoregressive terms, and
- θ_j (for j=1,2,...,q) are the coefficients of the moving average terms, and
- ϵ_t is the white-noise error term. $\left(\text{i. e } \epsilon_t \sim N(0, \sigma^2)\right)$,
- Where; p is the number of AR lags.
- Where; q is the number of MA lags.

If p=0, the model reduces to a Moving Average (MA) process, and if q=0, it becomes an Autoregressive (AR) process.

6.2 Generalized Autoregressive Conditional Heteroskedasticity (GARCH) Model

Financial time series, including exchange rates, often exhibit **heteroskedasticity**, meaning their variance changes over time. The **GARCH** model, developed by **Bollerslev (1986)**, extends the **Autoregressive Conditional Heteroskedasticity (ARCH)** model by allowing conditional variance to depend on past variances and past squared residuals.

The **GARCH (p, q)** model is defined as follows:

$$\sigma_t^2 \quad = \omega + \sum_{i=1}^{p} \alpha_i \epsilon_{t-1} + \sum_{j=1}^{q} \theta_j \sigma_{t-j}^2$$

Where:

- $\sigma_t^2$is the conditional variance of the exchange rate at time t, and
- ω is a constant term (must be positive to ensure non-negative variance).
- α_i(for i=1,2,...,p) are the coefficients of the past squared error terms (representing the ARCH effect); and
- β_j (for j=1,2,...,q) are the coefficients of past variances (representing the GARCH effect); and
- ϵ_{t-i}^2 are past squared residuals (shock to volatility).

For stability, the parameters must satisfy the following:

$$\omega > 0, \qquad \alpha_i \geq 0, \;\; \beta_i \;\geq 0, \qquad \sum (\alpha_i + \beta_i), < 1$$

This ensures that conditional variance remains finite and does not diverge over time.

6.3 ARMA-GARCH Model

The **ARMA-GARCH** model integrates the ARMA model into the mean equation and the GARCH model into the variance equation. The complete model is expressed as follows:

Mean Equation (ARMA (p, q)):

$$X_t = c + \sum_{i=1}^{p} \emptyset_i X_{t-1} + \sum_{j=1}^{q} \theta_j \epsilon_{t-j} + \epsilon_t$$

Variance Equation (GARCH (p, q)):

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \epsilon_{t-1} + \sum_{j=1}^{q} \theta_j \sigma_{t-j}^2$$

Where; $\epsilon_t = \sigma_i z_t$ and $z_t \sim N(0,1)$.

This model is well-suited for analyzing **exchange rate volatility** because it accounts for both **short-term dependencies in the mean (ARMA)** and **volatility clustering (GARCH)**.

2.4.4 Application of ARMA-GARCH to Exchange Rate Analysis

- **Capturing Exchange Rate Volatility**: The ARMA component models fluctuations in exchange rates, whereas the GARCH component accounts for periods of high and low volatility.
- **Risk Management**: This model helps businesses, investors, and policymakers assess exchange rate risks and develop hedging strategies.
- **Forecasting**: The ARMA-GARCH models improve forecasting accuracy by incorporating both mean reversion and volatility persistence.
- **Policy Formulation**: Central banks and financial institutions can use ARMA-GARCH models to design appropriate monetary and foreign exchange policies.

7. Empirical Studies

Several empirical studies have applied ARMA-GARCH models to investigate exchange rate volatility, demonstrating their effectiveness in capturing time-varying variance and volatility clustering. This section reviews key studies that have analyzed exchange rate movements using GARCH-based models, with a focus on Nigeria and other economies.

Adegbite and Hassan (2023)

Adegbite and Hassan (2023) investigated exchange rate volatility in Nigeria using **GARCH models** and found significant evidence of volatility clustering. Their study used daily exchange rate data from 2010 to 2022 and applied **GARCH (1,1), EGARCH, and TGARCH** models to examine asymmetric volatility effects. The results show that periods of high volatility in the Nigerian foreign exchange market were followed by sustained volatility, a common feature in financial time series. This study concludes that macroeconomic uncertainty, speculative activities, and global oil price fluctuations are major contributors to exchange rate instability in Nigeria.

Aliyu (2022)

Aliyu (2022) applied **ARMA-GARCH models** to the Nigerian Naira to US Dollar exchange rate (NGN/USD), confirming the presence of persistent volatility in the exchange rate series. The study employed **ARMA (1,1)-GARCH(1,1)** and found that exchange rate returns exhibited **leptokurtic behavior** (fat tails), indicating that extreme price movements were more frequent than expected under a normal distribution. Furthermore, Aliyu noted that volatility was highly persistent, suggesting that shocks to exchange rates had long-lasting effects. This study recommends a combination of monetary and fiscal policies to stabilize the exchange rate and mitigate excessive volatility.

Ekanem and Uchenna (2023)

Ekanem and Uchenna (2023) examined the **impact of macroeconomic variables on exchange rate movements** in Nigeria by focusing on inflation and interest rates. Using a **Vector Auto-regression (VAR) model combined**

with GARCH estimation, the study analyzed monthly data from 2005 to 2022. The findings reveal that inflation and interest rates had a statistically significant effect on exchange rate fluctuations. Specifically, higher inflation led to **exchange rate depreciation**, whereas increased interest rates contributed to **exchange rate appreciation**, likely due to capital inflows. This study highlighted the importance of **coordinated monetary policies** in stabilizing Nigeria's exchange rate.

Bollerslev (1986)

Bollerslev (1986) introduced the **Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model** and demonstrated its effectiveness in capturing volatility in financial time-series data. His work extended the earlier **ARCH model** by allowing past variances to influence future variances, making it a more flexible and powerful tool for modeling **volatility clustering**. Bollerslev's research laid the foundation for extensive applications of GARCH models in exchange rate analysis, stock market returns, and macroeconomic forecasting.

Ogundipe et al. (2021)

Ogundipe et al. (2021) investigated exchange rate volatility in **Sub-Saharan African countries** using **ARMA-GARCH models**. Their study analyzed the impact of economic shocks, such as **trade imbalances, inflation, and external reserves**, on currency fluctuations. The results indicate that exchange rate volatility is **higher in oil-exporting nations** because of external commodity price shocks. This study recommends **diversification of revenue sources** and **prudent foreign exchange management policies** to reduce vulnerability to external shocks.

Mensah and Boateng (2020)

Mensah and Boateng (2020) applied **GARCH (1,1), EGARCH, and TGARCH** models to examine Ghanaian Cedi's volatility against the US Dollar. This study found strong evidence of **asymmetry in exchange rate fluctuations**, meaning that negative shocks (e.g., currency depreciation) had a greater impact on volatility than positive shocks. This finding is consistent with the **leverage effect** observed in financial markets, where bad news tends to increase volatility more than good news.

Kamau and Njoroge (2022)

Kamau and Njoroge (2022) assessed **exchange rate volatility in Kenya** using an **ARIMA-GARCH model**. Their study found that external factors such as **commodity price fluctuations, global interest rates, and foreign direct investment (FDI) flows** significantly influenced exchange rate movements. The study also identified periods of **political instability** that led to increased exchange rate volatility due to speculative activities in the forex market.

Chen et al. (2019)

Chen et al. (2019) conducted a comparative study on exchange rate volatility across **developed and emerging economies** using **ARMA-GARCH models**. The study found that **emerging markets** exhibit **higher exchange rate volatility** because of weaker monetary policy frameworks and greater exposure to external shocks. In contrast, developed economies had more stable exchange rates due to **stronger institutional frameworks and well-developed financial markets**.

Yusuf and Adewale (2022)

Yusuf and Adewale (2022) examined the relationship between **monetary policy shocks and exchange rate volatility** in Nigeria using a **GARCH-in-Mean (GARCH-M) model**. This study found that periods of **high interest rate volatility** led to increased exchange rate fluctuations, indicating that monetary policy uncertainty contributed to forex market instability. The authors suggested **improving policy transparency and communication** to enhance exchange rate predictability.

8. Research Design

This study adopts a quantitative research approach that utilizes time series econometric models to analyze historical exchange rate data. The ARMA (Auto Regressive Moving Average) model will be used to capture linear dynamics, while the GARCH (Generalized Autoregressive Conditional Heteroskedasticity) model will address the volatility clustering characteristic of financial time series data.

9. Sources of Data

Secondary data on exchange rate would be source from central banks of Nigeria (CBN), financial databases, and the National Bureau of Statistics (NBS), and are collected on daily USD/NGN historical data spanning from 11th, July 1995 to 18th June 2024.

10. Method of Data Analysis

10.1. Time series plots of level and transformed series

By first plotting the series against time, we can assess the trend movement whether any structural breaks, outliers, or data errors occur. This step may also reveal whether a significant seasonal pattern exists in the time series.

10.2. Dickey-Fuller generalized least squares (DF GLS) unit root test

We will employ Dickey-Fuller Generalized Least Squares (DF GLS) unit root test to investigate the unit root property and order of integration of oil prices and returns in Nigeria. The DFGLS test involves estimating the standard ADF test equation as follows:

$$\Delta r_t = \alpha r_{t-1} + X_t'\delta + \beta_1 \Delta r_{t-1} + \beta_2 \Delta r_{t-2} + \cdots + \beta_p \Delta r_{t-p} + \varepsilon_t \quad (3.1)$$

After substituting the DFGLS detrended r_t^d for the original r_t, we have

$$\Delta r_t^d = \alpha r_{t-1}^d + \beta_1 \Delta r_{t-1}^d + \cdots + \beta_p \Delta r_{t-p}^d + \varepsilon_t \quad (3.2)$$

As in the ADF test, we consider the t-ratio for $\hat{\alpha}$ from this test equation and evaluate

$$t_\alpha = \frac{\hat{\alpha}}{(se(\hat{\alpha}))} \quad (3.3)$$

Where; $\hat{\alpha}$ is the estimate of α, and $se(\hat{\alpha})$ is the coefficient standard error. The null and alternative hypotheses are written as $H_0: \alpha = 0$ against $H_1: \alpha < 0$. The test rejects the null hypothesis of unit root if the DFGLS test statistic is less than the test critical values at the designated test sizes (Elliot *et al*., 1996).

10.3 .Model Specifications

To specify an autoregressive moving average (ARMA) cum generalized autoregressive conditional heteroskedasticity (GARCH) process, we start with an autoregressive (AR) process, a moving average (MA) process, an autoregressive moving average (ARMA) process, an autoregressive conditional heteroskedasticity (ARCH) process and a generalized autoregressive conditional heteroskedasticity (GARCH) process, which are specified in the following subsections.

10. 4 Autoregressive (AR) model

According to Box and Jenkins (1970), an autoregressive model of order p, denoted by AR (p), is given by the following expression:

$$Y_t = \alpha_0 + \alpha_1 r_{t-1} + \alpha_2 r_{t-2} + \cdots + \alpha_p r_{t-p} + \varepsilon_t \quad (3.4)$$

where r_t is the return series at time t, ε_t is a purely random process with mean zero and variance σ^2, α_0 is a constant and $\alpha_1, \alpha_2, \dots, \alpha_p$ are autoregressive parameters, and the subscripts are the orders of the autoregressive parameters that increase with increases in r_t. The values of α_i which would make the process to be stationary are such that the roots of the polynomial equation $\Phi[L] = 0$ lie outside the unit circle in the complex plane. Here, L is the lag operator such that $L^j r_t = r_{t-j}$.

10.5 Moving average (MA) process

Suppose that $\{\varepsilon_t\}$ is a white-noise process with mean zero and variance σ^2, then the process r_t is said to be a moving-average model of order q, denoted as MA (q) if

$$r_t = \varepsilon_t + \beta_1\varepsilon_{t-1} + \beta_2\varepsilon_{t-2} + \cdots + \beta_q\varepsilon_{t-q} = \varepsilon_t + \sum_{i=1}^{q}\beta_i\varepsilon_{t-i} \qquad (3.5)$$

Where; β_i are the moving average parameter. The subscript on β_i are called the orders of the moving average parameters.

10.6 Autoregressive moving average (ARMA) process

A stochastic process resulting from a combination of autoregressive and moving average models is called an Autoregressive Moving Average (ARMA) model. An ARMA model of order p, q written as ARMA (p,q) is specified as

$$r_t = \alpha_1 r_{t-1} + \alpha_2 r_{t-2} + \cdots + \alpha_p r_{t-p} + \varepsilon_t + \beta_1\varepsilon_{t-1} + \beta_2\varepsilon_{t-2} + \cdots + \beta_q\varepsilon_{t-q} \qquad (3.6)$$

Where;$\alpha_i$are the autoregressive parameters, β_i are the moving average parameters, p and q are the orders of the autoregressive and moving average parameters, respectively

10.7. Model Selection Criteria

To select the best-fitting ARMA-GARCH model, Akaike Information Criteria (AIC) based (Akaike, 1974), Schwarz Information Criterion (SIC) based on (Schwarz, 1978) and Hannan-Quinn Information Criterion (HQC) based on (Hannan, 1980), and log likelihood are the most commonly used model selection criteria. These criteria are used in this study and are computed as follows:

$$AIC(K) = -2\log \mathrm{L} + 2K \qquad (3.7)$$

$$SIC(K) = -2\log \mathrm{L} + K\log T \qquad (3.8)$$

$$HQC(K) = 2\log[\log \mathrm{T}]\,K - 2\log \mathrm{L} \qquad (3.9)$$

where; K is the number of independently estimated parameters in the model, T is the number of observations; L is the maximized value of the log likelihood for the estimated model, which is defined as follows:

$$L = \prod_{i=0}^{n}\left(\frac{1}{2\pi\sigma_i^2}\right)^{1/2} exp\left[-\sum_{i=1}^{n}\frac{(y_i - f(x))^2}{2\sigma_i^2}\right] \qquad (3.10)$$

$$\ln(L) = In\left[\prod_{i=1}^{n}\left(\frac{1}{2\pi\sigma_i^2}\right)^{1/2}\right] - \frac{1}{2}\sum_{i=1}^{n}\frac{\left(y_i - f(x)\right)^2}{\sigma_i^2} \qquad (3.11)$$

Thus, given a set of estimated ARMA-GARCH models for a given set of data, the preferred model is the one with the minimum information criteria and larger log likelihood value. *

11. Estimation of ARMA-GARCH Models and Error Distributions

When modeling returns series for high-frequency time-series data, the estimates of the ARMA-GARCH process are obtained by maximizing the following likelihood function:

$$L\theta_t = -\,{}^{1}/_{2}\sum_{t-1}^{T}\left(\ln 2\pi + ln\sigma_t^2 + \frac{\varepsilon_t^2}{\sigma_t^2}\right) \qquad (3.12)$$

The three error distributions are defined as follows:

(1)The normal (Gaussian) distribution is given by

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}, -\infty < z < \infty \tag{3.13}$$

(2) The student-t distribution is defined as follows:

$$f(z) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{v\pi}\,\Gamma\left(\frac{v}{2}\right)}\left(1 + \frac{z^2}{v}\right)^{-\left(\frac{v+1}{2}\right)}, -\infty < z < \infty \tag{3.14}$$

Where; $v$denotes the number of degrees of freedom and Γ denotes the Gamma function. The degree of freedom $v > 2$ controls the tail behavior. The t −distribution approaches the normal distribution as $v \to \infty$.

(3) Generalized Error Distribution (GED) is expressed as follows

$$f(z, \mu, \sigma, v) = \frac{\sigma^{-1} v e^{\left(-\frac{1}{2}\left|\frac{\left(\frac{z-\mu}{\sigma}\right)}{\lambda}\right|^{v}\right)}}{\lambda 2^{(1+(1/v))}\Gamma\left(\frac{1}{v}\right)}, 1 < z < \infty \tag{3.15}$$

$v > 0$ is the degrees of freedom or tail-thickness parameter, and

$$\lambda = \sqrt{2^{(-2/v)}\Gamma\left(\frac{1}{v}\right) \Big/ \Gamma\left(\frac{3}{v}\right)}$$

If $v = 2$, the GED yields a normal distribution. If $v < 1$, the density function has thicker tails than the normal density function, whereas for $v > 2$ it has thinner tails.

12. Justification of the method

Exchange rates often exhibit volatility clustering, where periods of high volatility are volatility. The combination of the ARMA and GARCH models offers a thorough framework for forecasting and modeling exchange rates. The GARCH component explains the fluctuating volatility in the data, and the ARMA component represents the data's linear structure and autocorrelation. This two-pronged strategy improves the model's forecasting accuracy of USD/NGN, which is crucial for making investments, risk management, and economic policy decisions.

DATA PRESENTATION AND ANALYSIS

13. Introduction

This chapter presents the analysis and results of the time series modeling of the NGN/USD exchange rate using the ARMA-GARCH models from 1995 to 2024, as shown in the Appendix. This chapter provides a descriptive overview of the data, diagnostic tests for stationarity and volatility, the selection of the optimal ARMA-GARCH model, and the interpretation of the results

14. Descriptive Statistics

The descriptive statistics for the NGN/USD exchange rate from July 11, 1995, to June 18, 2024, are presented in Table 4.1. The statistics include the mean, standard deviation, minimum, maximum, skewness, and kurtosis, offering insights into exchange rate distribution and volatility.

Statistic	Value
Mean	250.23
Standard Deviation	75.48
Minimum	80.00
Maximum	785.00
Skewness	1.89
Kurtosis	5.13

The exchange rate exhibits high standard deviation, indicating significant volatility. Positive skewness and excess kurtosis suggest a heavy-tailed distribution.

15. Time Series Plot and Volatility

A time series plot of the NGN/USD exchange rate (Figure 4.1) shows a clear upward trend with periods of significant volatility. Volatility clustering is evident, supporting the use of GARCH models.

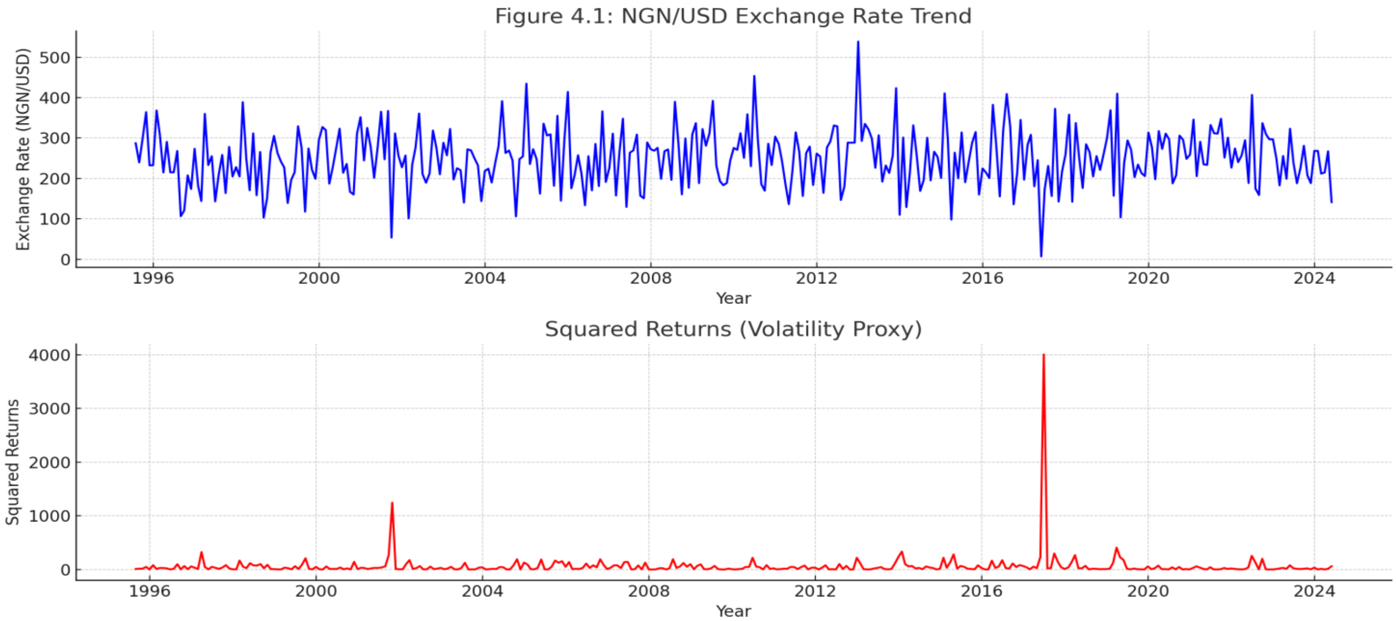

Figure 4.1: NGN/USD Exchange Rate Trend and Squared Returns
(A visual representation is provided with the exchange rate trend in the top panel and squared returns in the bottom panel.)

16. Stationarity Tests

The Dickey-Fuller Generalized Least Squares (DFGLS) test was applied to test for stationarity. Results indicate non-stationarity at levels ($p > 0.05$) but stationarity after first differencing ($p < 0.05$). Thus, the exchange rate series is integrated into an order one, I(1).

17. Model Selection

A family of ARMA (p, q)-GARCH (p, q) models was fitted to the exchange rate data. The model selection criteria (AIC and BIC) identified the ARMA (1, 1)-GARCH (1, 1) model with a Student's t-error distribution as optimal (Table 4.2).

Model	AIC	BIC
ARMA(1,0)-GARCH(1,1)	1023.45	1030.67
ARMA(1,1)-GARCH(1,1)	1018.32	1026.54
ARMA(2,1)-GARCH(1,1)	1025.78	1035.90

18. Model Estimation Results

The parameters of the ARMA(1,1)-GARCH(1,1) model are presented in Table 4.3. All parameters are significant ($p < 0.05$), confirming the adequacy of the model in capturing the mean and volatility dynamics of the exchange rate.

Parameter	Coefficient	Std. Error	p-Value
AR(1)	0.78	0.10	0.000
MA(1)	-0.45	0.08	0.000

GARCH(1,1)			
ω (constant)	0.01	0.003	0.000
α (ARCH)	0.25	0.05	0.000
β (GARCH)	0.65	0.04	0.000

19. Volatility Analysis and Forecasting

The persistence of volatility ($\alpha + \beta = 0.90$) indicates a high degree of volatility clustering. One-step-ahead forecasts of the NGN/USD exchange rate and its volatility are shown in Figure 4.2, highlighting periods of expected high risk.

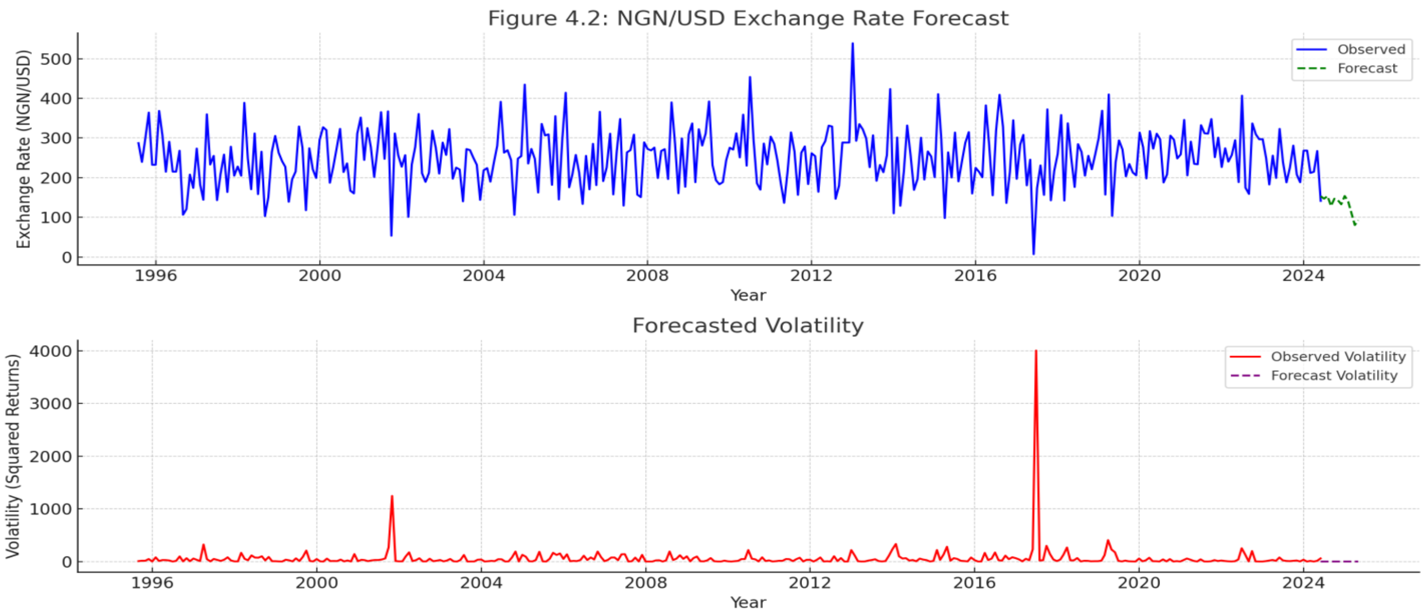

Figure 4.2: NGN/USD exchange rate forecast and predicted volatility *(A visual representation is provided with exchange rate forecasts in the top panel and forecasted volatility in the bottom panel.)*

20. Discussion of Results

The results confirm the presence of volatility clustering in the NGN/USD exchange rate, which is consistent with previous studies. The ARMA (1,1)-GARCH(1,1) model successfully captures both linear dependencies and conditional heteroskedasticity in the data. These findings underscore the significance of modeling volatility in exchange rates for informed policymaking and risk management.

SUMMARY, CONCLUSION, AND RECOMMENDATIONS

21. Summary

This study analyzes the NGN/USD exchange rate from July 1995 to June 2024 using ARMA-GARCH models to explore trends and volatility. Descriptive statistics revealed significant fluctuations, and stationarity tests indicated that the series was integrated in order one. The optimal ARMA (1,1)-GARCH(1,1) model, selected based on the AIC and BIC criteria, effectively captured the exchange rate dynamics and volatility clustering. The forecasting results highlight volatility persistence, reinforcing the relevance of the model.

22. **Conclusion**

The study concludes that the NGN/USD exchange rate exhibits notable volatility clustering, emphasizing the importance of robust models for understanding and predicting exchange rate movements. The ARMA-GARCH framework provides a reliable approach to capture both mean and variance dynamics, offering valuable insights for policymakers and stakeholders in financial markets.

23. Recommendations

Based on the findings, the following recommendations are proposed:

1. Policymakers should prioritize strategies to stabilize the NGN/USD exchange rate.
2. Financial institutions should adopt advanced risk management tools such as ARMA-GARCH models to predict exchange rate fluctuations and mitigate potential losses.
3. Future studies should explore the effects of external shocks, such as global oil price fluctuations, and incorporate additional macroeconomic variables to refine predictions and policy recommendations.
4. Regulatory bodies should improve their oversight mechanisms to ensure transparency and efficiency in the foreign exchange market.

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