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INVESTIGATING SOME MACRO ECONOMIC VARIABLES IN NIGERIA USING ARIMAX AND ARIMA MODELS

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Abstract

This study investigates the behavior of inflation in Nigeria by modeling and forecasting its dynamics using Autoregressive Integrated Moving Average (ARIMA) and its extension with exogenous variables (ARIMAX) from 1990 to 2023. Inflation is modeled alongside key macroeconomic indicators: exchange rate, interest rate, and unemployment rate. Time series techniques were employed, including unit root testing, transformation using natural logarithms, and fitting of optimal ARIMA and ARIMAX models. The ARIMA (0, 0, 1) and ARIMAX (0,0,1) models were identified as best-fitting models for inflation forecasting. Although both models showed statistical adequacy with normally distributed residuals and no significant autocorrelation, ARIMA outperformed ARIMAX in terms of in-sample forecast accuracy with lower RMSE and MAE values. However, the ARIMAX model provided insights into the role of unemployment as a significant negative predictor of inflation. This study concludes that while ARIMA provides better short-term forecasts, ARIMAX offers a richer understanding of the inflation process by incorporating macroeconomic variables. These findings offer valuable input for monetary policy planning and economic modeling in Nigeria.

INTRODUCTION

A time series is an ordered sequence of observations. Although the ordering is usually performed through time, particularly in terms of some equally spaced time intervals, the ordering may also be performed through other dimensions, such as space (Adenomon, 2017). Time series occur in a variety of fields, examples are in engineering, geophysics, business, economics, medical studies, meteorology, quality control, social sciences, and agriculture. The list of areas cannot be exhaustive.

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There are various objectives to studying time series data. These include understanding and description of the generated mechanism, the modeling of future values, and optimum control of the system. The uses of time series analysis are performed in Equation (i). It helps in the analysis of the past behavior of a variable, (ii) it helps in modeling, and (iii). It helps in evaluation of current achievement (iv).It helps in making comparative studied. Therefore, the body of statistical methodology available for analyzing time series is referred to as time series analysis (Cooray, 2008).

Modeling univariate time series is very useful for forecasting such series. Over the years, Autoregressive Moving Average (ARMA) and Autoregressive Integrated Moving Average (ARIMA) models have become popular and are excellent for modeling univariate time-series data, as proposed by Box and Jenkins (1970), and its extension with exogenous variables as Autoregressive Integrated Moving Average with Explanatory Variable (ARIMAX) is becoming popular because researchers have found that the ARIMAX model can outperform the ARMA or ARIMA models (Kongcharoen and Kruangpradit, 2013). These models are applied in almost all fields of endeavors such as engineering, geophysics, business, economics, finance, agriculture, medical sciences, social sciences, meteorology and quality control etc. (Kirchgassner and Wolters, 2007). This project considered modeling inflation using Exchange, Interest and unemployment rates as exogenous variables with the ARIMAX model. Inflation, exchange, interest, unemployment, and growth rates are the big macroeconomic issues of our time (Lipsey and Chrystal, 1999). Inflation is bad, especially when unexpected, because it distorts the working of the price system, creates arbitrary redistribution from debtors to creditors, creates incentives for speculative rather than productive investment activity, and is usually costly to eliminate. Inflation is defined as a positive growth rate of the general price level. Eitrheim et al. (2004) noted that if the inflation modeling is sufficiently close to the target, policy instruments (a short-term interest rate) are left unaltered. However, if the modeling rate of inflation is higher or lower than the target, the monetary instrument will be changed until the revised forecast is close to the inflation target. Because Inflation, exchange, interest, unemployment, and growth rates can affect (either positive or negative), these macroeconomic variables are of great interest to Central Banks of many countries of the world.

Statement of the Problem

In recent years, the Nigerian economy has experienced an economic meltdown, leading to high inflation, exchange, unemployment, and interest rates, causing gross domestic growth to decline. These situations affected both the rich and poor, and the employed and unemployed. In fact, everyone felt the negative effects of these macroeconomic variables (inflation, exchange, unemployment and interest rates). From economic theory point of view, inflation, exchange, unemployment, and interest rates are interrelated, for instance, inflation and unemployment rates (i.e Phillips Curve) and other variables (exchange and interest rates) with inflation rate. In this project, we aim to study the effects of lags in the inflation rate, unemployment rate, exchange rate, and interest using the ARIMAX and ARIMA Models. Thereafter, inflation rates are modeled using ARIMAX and ARIMA models to examine whether the inflation rate out sample forecast can be improved using ARIMAX compared to the ARIMA model.

Objectives

The objectives of this study are as follows:

i. Investigate the behavior and performance of ARIMAX and ARIMA models under different data conditions;

ii. The efficiency of the models was compared under different data conditions

iii. Determine the best model among ARIMAX and ARIMA for forecasting.

The significance of the study

This study examined and forecasted inflation rates in Nigeria using the ARIMAX and ARIMA models. Hence, this study can benefit the following:

i. Exposed the effects and the interrelationship among inflation, exchange, unemployment, and interest rates, which will help the monetary policy marker of the Central Bank of Nigeria in decisions about inflation targets.

ii. This study will provide a useful resource for economics and statistics students and researchers.

iii. This project will also help local and international investors determine what to invest in in Nigeria.

iv. Also useful to the government is to know the real standard of living of its citizens.

Research Methodology

Research design

Techniques for Data Analysis for the research design to be used in this study aim to investigate macroeconomic variables using ARIMA and ARIMAX models for analysis. The research study used a secondary method as the instrument to draw data from the respondents.

Source of Method

The data sources form National Bureau of Statistics (NBS) and the Central Bank of Nigeria. The data used for analysis are from 1990 to 2023

Methods of Data Collection

The data to be used in this project will be Annual Data on Exchange Rate, Inflation Rate, Interest Rate, and Unemployment Rate. The inflation rate is the variable of interest (response variable), whereas the exogenous variables are Exchange Rate, Interest Rate, Unemployment Rate. The variables are transformed using the natural logarithm to ensure stability and normality and reduce skewness and variability.

Data Analysis Technique

The following are the methods of analysis:

Unit Root Test

Engle and Granger, (1987) considered seven test statistics in a simulation study to test co-integration. They concluded that the Augmented Dickey Fuller test was recommended and can be used as a rough guide for applied work. The essence of the unit root test is to avoid spurious regression.

To distinguish a unit root, the regression

$$\Delta Y_t = b_o + \sum_{j=1}^{k} b_j \Delta Y_{t-j} + \beta t + \gamma Y_{t-1} + u_t$$

The model in (1) may be run without t if a time trend is not necessary. This technique was applied by Ajayi and Mougoue (1996). If there is a unit root, differing Y should result in a white-noise series (no correlation with Y_{t-1}).

The Augmented Dickey-Fuller (ADF) test of the null hypothesis of no unit root test is of the form H_o: $\beta = \gamma = 0$ (if there is trend we use F-test) and H_o: $\gamma = 0$ (if there is no trend we use t-test). If the null hypothesis is accepted, we assume that there is a unit root and that the data are different before running the regression. If the null hypothesis is rejected, the data are stationary and can be used without differencing (Salvatore &Reagle, 2002).

ARIMA Model and Estimation

The ARIMA model is an approach that combines moving averages and autoregressive models (Dobre & Alexandru, 2008). The pioneers in this area were Box and Jenkins, who were popularly known as the Box-

Jenkins (BJ) methodology but technically known as the ARIMA methodology (Gujarati, 2003). The emphasis of these methods is not on constructing single-equation or simultaneous-equation models but on analyzing the probability, or stochastic, properties of economic time series on their own under the philosophy ‘let the data speak for themselves’. Unlike the regression models, in which Y_t is explained by a k regressor X_1, X_2 . . . X_k, the BJ-type time-series models allow Y_t to be explained by past or lagged values of y Y itself and stochastic error terms. For this reason, ARIMA Models are sometimes called theoretical models because they are not derived from economic theory.

The Box-Jenkins ARMA (p,q) model is a combination of the AR and MA models as follows: $y_t = a_o + a_1 y_{t-1} a_2 y_{t-2} + ... + a_p y_{t-p} - b_1 u_{t-1} - b_2 u_{t-2} - ... - b_q u_{t-q} + u_t$

Box and Jenkins recommend a difference non-stationary series of one or more times to achieve stationarity. This produces an ARIMA model, with the ‘I’ standing for ‘Integrated’. However, its first difference $\Delta y_t = y_t - y_{t-1} = u_t$ is stationary, so y is ‘Integrated of order 1’ or y~I(1).

There are three primary stages in building a Box-Jenkins time series model; they are model identification; model estimation, and model validation.

Theoretical patterns of ACF and PACF

Type of model	typical pattern of ACF	Typical pattern of PACF
AR(p)	decays exponentially or with damped sine-wave pattern or both	Significant spikes through lags p
MA(q)	Significant spikes through lag p	Declines exponentially
ARMA(p,q)	Exponential decay	Exponential decay

A test for adequacy of the fitted model is the chi-square test for goodness of fit, which is called the Ljung-Box test (Ljung& Box, 1978). This test assumes all residual ACFs as a set. The test statistic is given as $Q = n(n+2)\sum_{i=1}^{k}(n-i)^{-1}\gamma_i^2(\hat{a})$ where $\gamma_i^2(\hat{a})$ is the estimate for $\rho_j(\hat{a})$ and n is the number of observations used to estimate the model. The statistic Q approximately follows a chi-squared distribution with k-v degrees of freedom, where v is the number of parameters estimated in the model. If we accept the null hypothesis, then the fitted model is adjudged to be adequate.

ARIMAX Model

The ARIMA model is extended into an ARIMA model with an explanatory variable (X_t), called ARIMAX (p,d,q). Specifically, ARIMAX (p,d,q) can be represented by

$\phi(L)(1-L)^d Y_t = \Theta(L) X_t + \theta(L)\varepsilon_t$

Where L is the lag operator, d=difference order, p is the AR order, q is the MA order, explanatory variables (X_t) and ε_t is the error term while ϕ, Θ, θ are the coefficients of the AR, MA and exogenous variables ((Kongcharoen and Kruangpradit, 2013)

Modeling Assessment

The following criteria were used for modeling assessments:

1. The mean absolute error (MAE) has a formular $MAE_j = \frac{\sum_{i=1}^{n}|e_i|}{n}$. This criterion measures deviations from the series in absolute terms, and measures how much the modeling is biased. This measure is one of the most common ones used to analyze the quality of different forecasts.

2. The Root Mean Square Error (RMSE) is given as $RMSE_j = \sqrt{\frac{\sum_i^n (y_i - y^f)^2}{n}}$ where y_i is the time series data and y^f is the modeling value of y (Caraiani, 2010)

For the two measures above, the smaller the value, the better the fit of the model (Cooray, 2008).

Incorporating the Model

These are parameters of interest when fixing the model using regression analysis.

.$Y_i = F(X_i, \beta) + e_i$

Y_i =Function

X_i = Independent Variable

β_i = Unknown Parameters

ϵ_i = Error terms

Multiple Parameters of Medel

$\gamma = \beta_o + \beta_1 \times_i + \beta_2 \times_2 + \ldots + \beta_q \times_q + \epsilon$

$\beta_o = \bar{Y} - \beta_1 \times_1 - \beta_2 \overline{X}_2$

$\beta_1 = \frac{(\sum X^2{}_2)(\sum X_1 Y) - (\sum X_1 X_2)(\sum X_2 Y)}{(\sum X^2{}_1)(\sum X^2{}_2) - (\sum X_1 X_2)^2}$

$\beta_2 = \frac{(\sum X^2{}_1)(\sum X_2 Y) - (X_1 X_2)(X_1 Y)}{(\sum X^2{}_1)((\sum X^2{}_2)) - (X_1 X_2)^2}$

Whileβ_0, β_1, β_2&β_3 are the parameters of the model $\in$ *is distrubance*.

DATA PRESENTATION

The data used in this project are presented in tables and figures.

Table 1: Annual Exchange Rate Data, Inflation Rate, Interest Rate, and Unemployment Rate

Year	Exchange rate	Inflation rate	Interest Rate	Unemployment Rate
1990	8.038	7.5	25.5	3.5
1991	9.909	12.9	20.01	3.1
1992	17.298	44.5998	29.8	3.4
1993	22.051	57.1998	18.3199	2.7
1994	21.886	56.9999	20.9999	2
1995	21.886	72.9001	20.18	1.8
1996	21.886	30.3999	19.7401	3.4
1997	21.886	8.2	13.54	3.2
1998	21.886	10.3	18.2899	3.2
1999	92.694	6.7	21.3201	3
2000	102.105	6.9	17.98	18.1
2001	111.943	18.9	18.2899	13.7
2002	120.97	12.9	24.8501	12.2
2003	129.356	14	20.7101	14.8
2004	133.5	14.9	19.18	11.8
2005	132.146	17.9	17.95	11.9
2006	128.652	8.2	17.26	13.7
2007	125.834	5.3	16.94	14.6
2008	118.567	11.6001	16.94	14.9
2009	148.88	13.7001	15.14	19.7
2010	150.298	10.8	18.99	21.1
2011	153.861	10.3	17.59	23.9
2012	157.499	11.5	16.02	24.3

2013	157.312	8.5	17.0899	28.5
2014	158.553	8.05	16.28	30
2015	192.439	8.2	16.8599	24
2016	253.489	9.6	16.54	25
2017	305.8	16.5	17.58	16.5
2018	361.41	11.61	19.33	23.1
2019	306	11.98	15.53	5.21
2020	380.25	13.25	12.32	33.3
2021	399	15.63	11.48	5.39
2022	448	21.34	12.33	5.3
2023	688.7	28.92	14.01	5.4

Source: CBN/ NBS Statistical Bulletin 2023

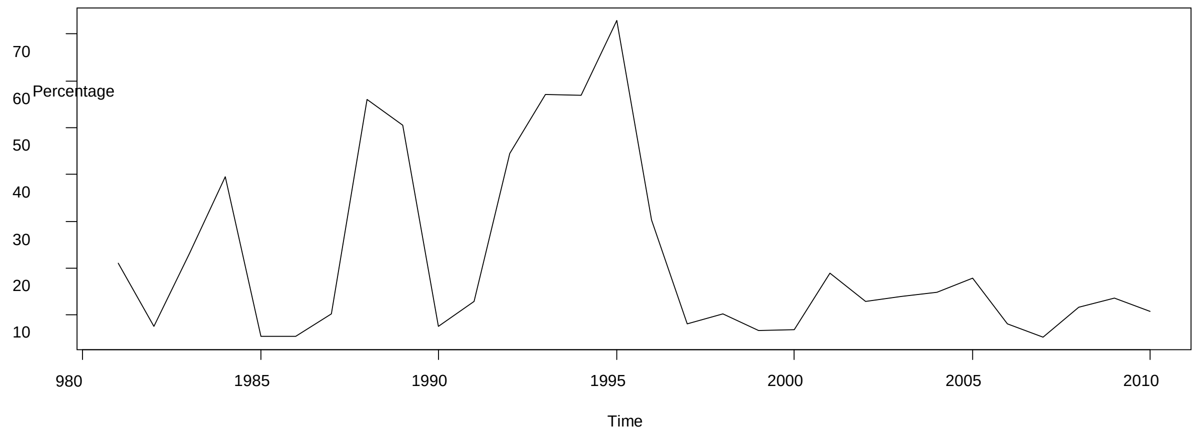


Fig. 1: Plot of Inflation Rate in Nigeria from 1990 to 2025

Figure 1 shows the inflation rate in Nigeria from 1981 to 2010. Nigeria experienced inflation from 1993 to 1996. Inflation rates were low from 2000 to 2010.

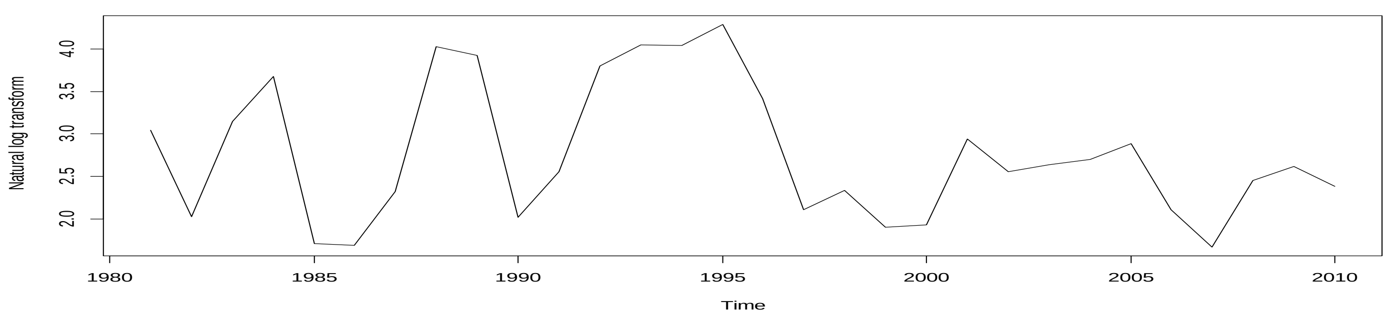


Fig 2: Plot of Natural Log transform of Inflation Rate from 1981 to 2010

Figure 2 shows the natural log-transform of the inflation rate in Nigeria from 1981 to 2010. Nigeria experienced inflation from 1993 to 1996. Inflation rates were low from 2000 to 2010. In addition, there is a reduction in the inflation rate trend after transformation.

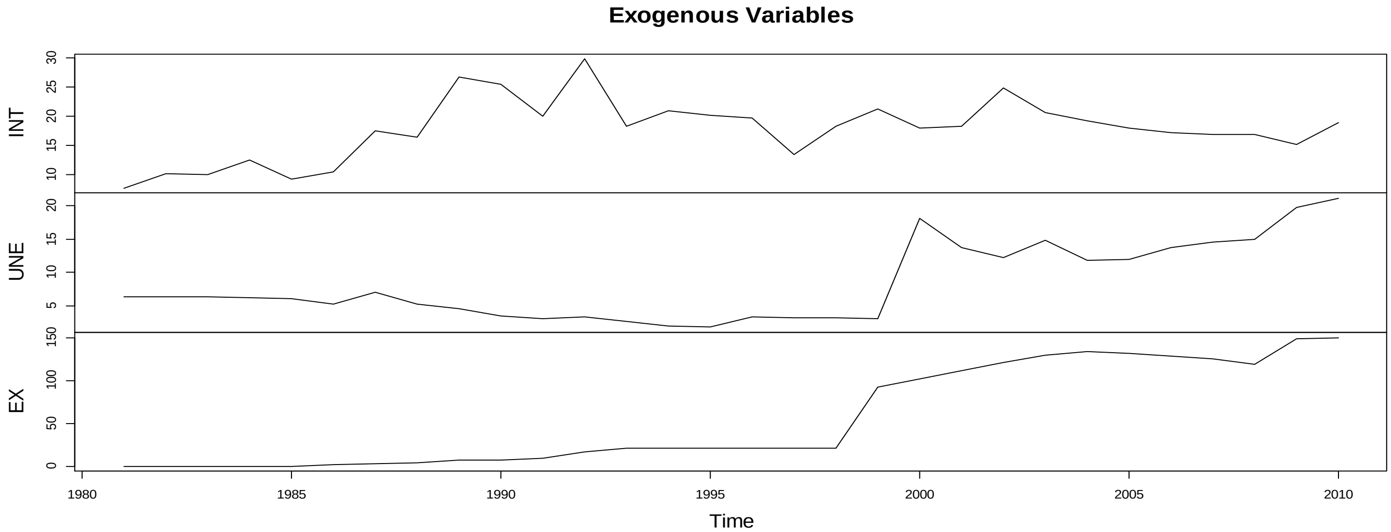


Fig 3: The Plots of Interest Rate (INT), Unemployment Rate (UNE), and exchange rate (EX) from 1981 to 2010.

Figure 3 presents the plots of Interest Rate (INT), Unemployment Rate (UNE), and exchange rate (EX) from 1981 to 2010 in Nigeria. The interest rate shows a decrease from 2002 to 2010, but unemployment and exchange rates show an increase from 2002 to 2009. This unemployment and exchange rate situation will definitely affect the standard of living in Nigeria if not properly controlled.

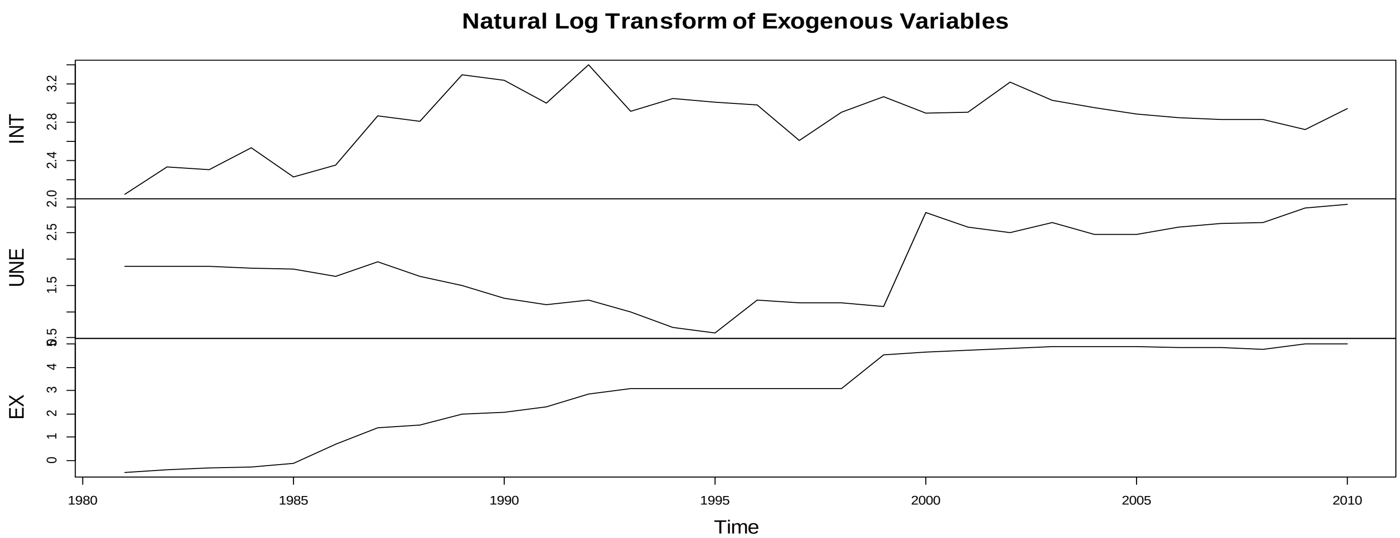


Fig 4: The Plots of the Natural Transform of Interest Rate (INT), Unemployment Rate (UNE), and Exchange Rate (EX) from 1981 to 2010

Figure 4 presents the plots of the natural log transform of Interest Rate (INT), Unemployment Rate (UNE), and exchange rate (EX) from 1981 to 2010 in Nigeria. The log of interest rate shows a decrease from 2002 to 2010, but for logs of unemployment and exchange rates, it increases from 2002 to 2009. This unemployment and exchange rate situation will definitely affect the standard of living in Nigeria if not properly controlled. This similar to figure 3 above.

DATA ANALYSIS AND RESULTS

The data analysis of this dissipation was carried out in an R software environment using the tseries and TSA packages.

1. Descriptive Statistics:

These numbers provide key insights into the distribution of the data for each variable:

- **Mean:** Average value of data points for each variable.
  - **LNINF:** 2.616
  - **LNINTR:** 2.922
  - **LNUEMP:** 2.244
  - **LNEX:** 4.397
- **Median:** The middle value of the dataset when it's ordered.
  - **LNINF:** 2.451
  - **LNINTR:** 2.898
  - **LNUEMP:** 2.617
  - **LNEX:** 4.846
- **Maximum:** The highest value in the dataset.
  - **LNINF:** 4.289
  - **LNINTR:** 3.395
  - **LNUEMP:** 3.401
  - **LNEX:** 5.890
- **Minimum:**lowest value in the dataset.
  - **LNINF:** 1.668
  - **LNINTR:** 2.606
  - **LNUEMP:** 0.588
  - **LNEX:** 2.084
- **Standard Deviation (Std. Dev.):** A measure of the spread or variability of the data.
  - **LNINF:** 0.673
  - **LNINTR:** 0.161
  - **LNUEMP:** 0.927
  - **LNEX:** 1.088

2. The shape of the Distribution:

These values give us an idea of the shape of the data distribution, whether symmetric, skewed, or extreme.

- **Skewness:**
  - **LNINF:** 1.176 (Positively skewed, meaning the right tail is longer)
  - **LNINTR:** 0.946 (Moderately positively skewed)
  - **LNUEMP:** -0.446 (Negatively skewed, meaning the left tail is longer)
  - **LNEX:** -0.720 (Moderately negatively skewed)
- **Kurtosis:** A measure of the "tailedness" of the distribution. Higher kurtosis indicates more extreme values (outliers).
  - **LNINF:** 3.544 (Leptokurtic, slightly more outliers)
  - **LNINTR:** 4.396 (Leptokurtic, more outliers)
  - **LNUEMP:** 1.606 (Platykurtic, relatively fewer outliers)
  - **LNEX:** 2.199 (Mesokurtic, somewhat normal distribution)

3. Normality Tests:
The **Jarque-Bera** test assesses whether the data follow a normal distribution. A higher test statistic and a p-value below 0.05 suggest that the data significantly deviate from normality.

- **Jarque-Bera:**

(a) **LNINF:** 7.287 (p-value = 0.026), suggesting a deviation from normality.
(b) **LNINTR:** 6.915 (p-value = 0.032), indicating deviation from normality.
(c) **LNUEMP:** 3.423 (p-value = 0.181) – This is not significant, so the data could be normal.
(d) **LNEX:** 3.396 (p-value = 0.183). Similarly, this suggests no significant deviation from normality.

Other Information:

- **Sum:** The sum of all data points in the dataset for each variable.

(a) **LNINF:** 78.479
(b) **LNINTR:** 87.662
(c) **LNUEMP:** 67.321
(d) **LNEX:** 131.895

- **Sum of Squares of Deviations (Sum Sq. Dev.):** Measures the variability in the data. Higher values indicate higher variability.

(a) **LNINF:** 13.120
(b) **LNINTR:** 0.748
(c) **LNUEMP:** 24.945
(d) **LNEX:** 34.306

- **Observations:** number of data points (30 for each variable.

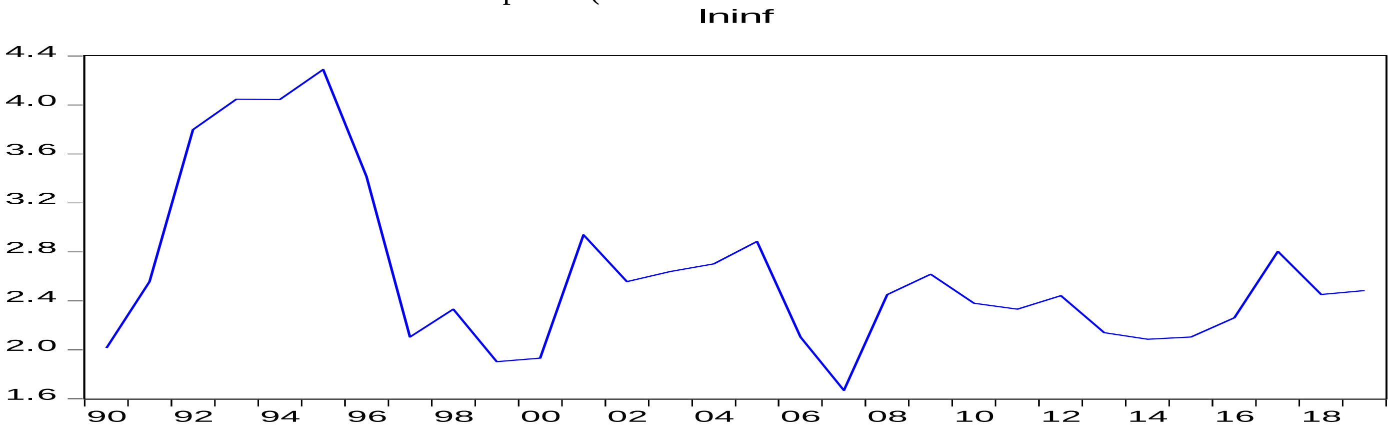


Figure 1i

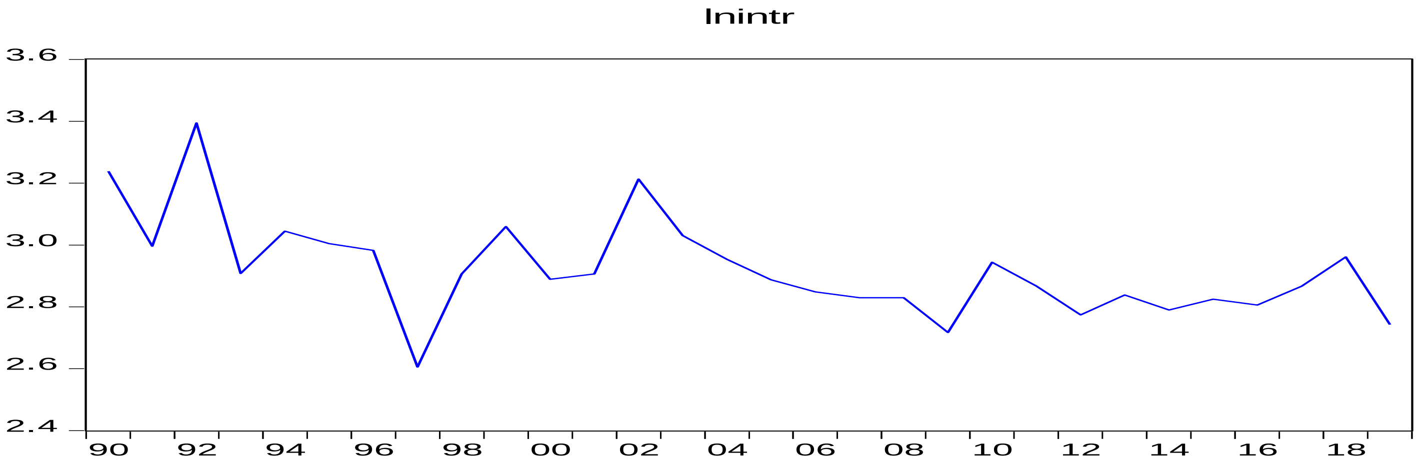


Figure 1ii

Figures i and ii (log of inflation and log of interest rates) show positive skewness, meaning they have longer right tails, (which mean there are more extreme values on the higher end) and their distributions seem leptokurtic, implying they peaked with fatter tails compared to a normal distribution. This indicates a higher likelihood of extreme values (outliers)

Outputs from the tests and model fitting

1. Augmented Dickey-Fuller (ADF) Test:

The Augmented Dickey-Fuller test is a statistical test used to determine whether a time series has a unit root, which indicates non-stationarity.

Variable	Test value	P-value	Order
Inflation Rate	-4.1834	0.01575	I(0)

This indicates that the inflation rate is stationary at a 5% significance level.

Table 2**: Automatic ARIMA Model** Optimal model: ARIMA (0, 0,1)

Variable	Inflation rate	Standard deviation	z-value	p-value
Intercept	0.9336	0.o565	16.5268	0.0000
MAI	0.999999	0.1028	9.7267	0.0000
RMSE	0.1572			
MAE	0.1238			
JB Test	P-value =0.6934			
Model adequacy	Log 12 (P-value=0.5632) Log 24 (P-value=0.6322)			

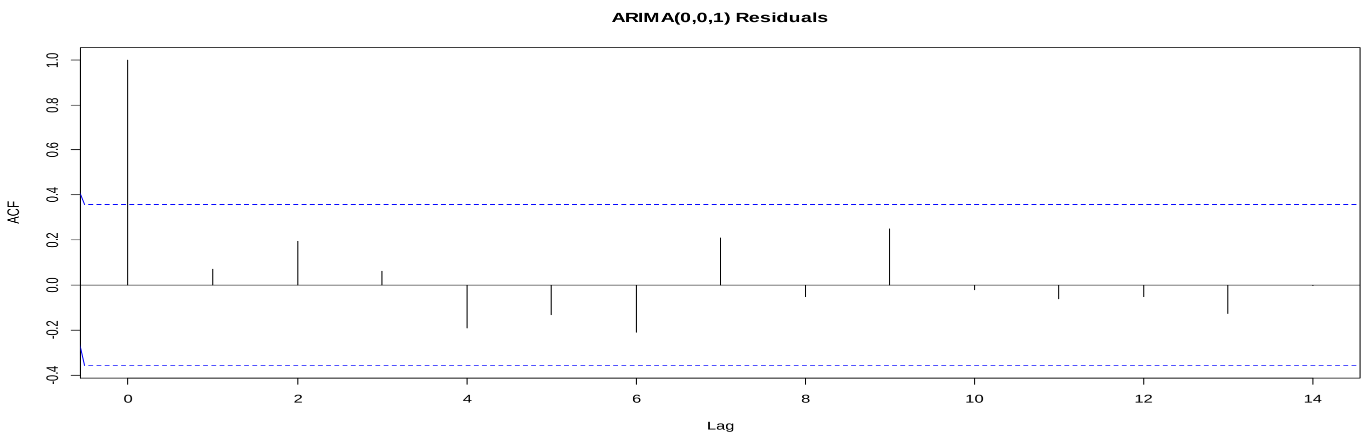


Figure 5

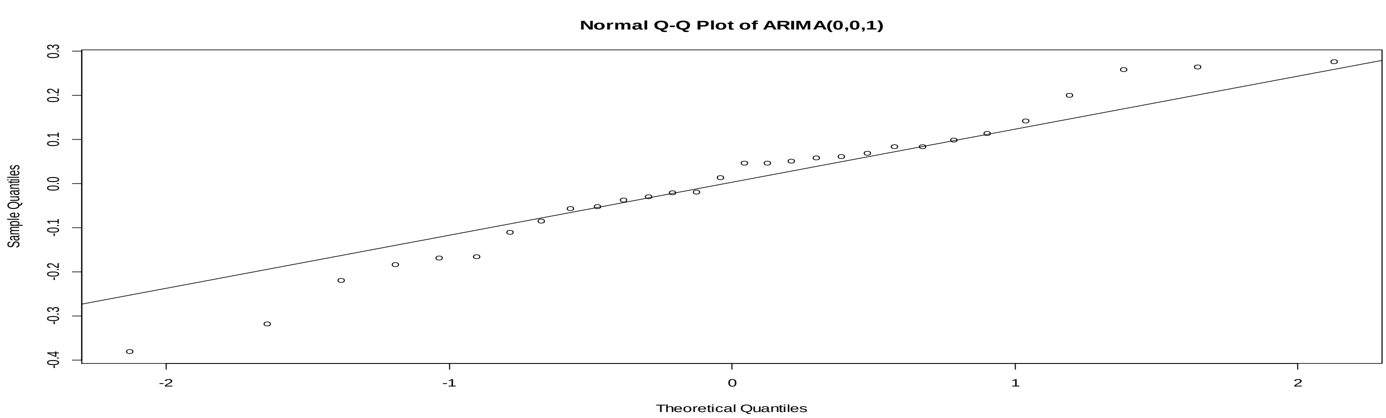


Figure 6 The optimal ARIMA model for inflation is ARIMA (0, 0,1), with parameters (intercept, MAI) that are significant (P-value < 0.05). The RMSE and MAE were 0.1238, respectively, whereas the JB test revealed that they were normally distributed (P-value =0.6938 < 0.05); thus, the model was adequate at log 12 and log 24.

AUTOMATIC ARIMAX MODEL
Optimal model: ARIMAX (0, 0,1)
Variable: Inflation Rate

Parameter	Estimate	Standard deviation	Z-Value	P-Value
Intercept	2.5691	1.2759	2.0136	0.04406
MAI	0.999995	0.1117	8.9533	0.0000
Inex	-0.0099	0.1384	-0.0717	0.9429
Inintr	0.2688	0.3163	0.8500	0.3953
Inuemp	-0.3174	0.1192	-2.6634	0.0077
RMSE	0.3768			
MAE	0.2732			
JB test	P-Value	Log 12(P-value=0.2822) Log 24(P-value=0.6956)		

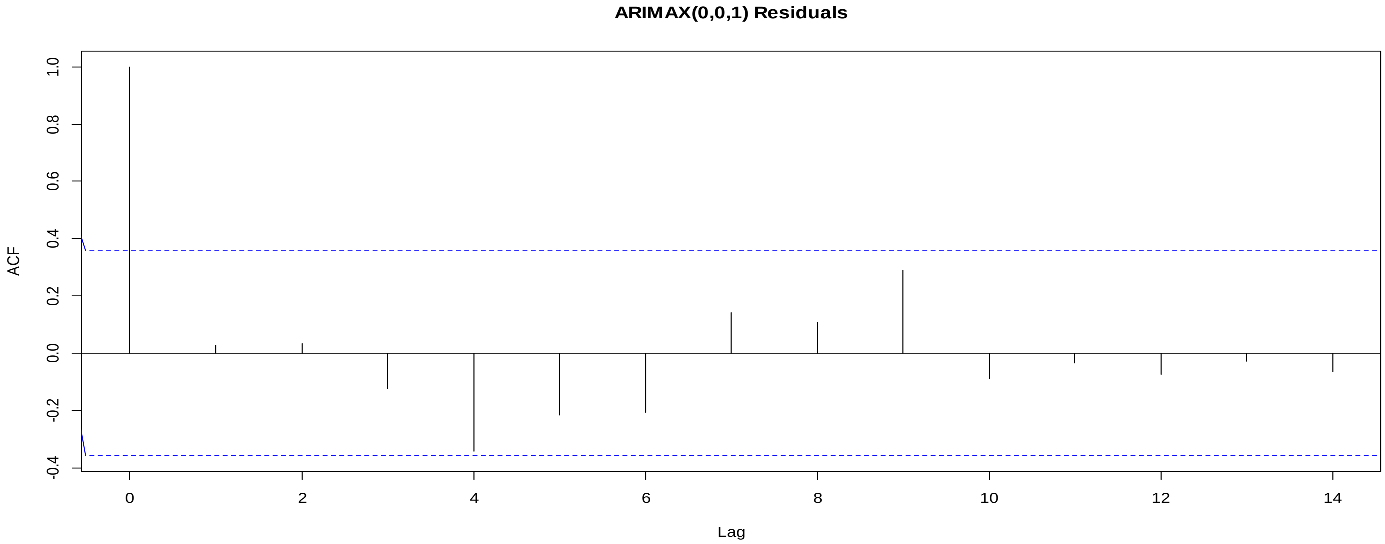
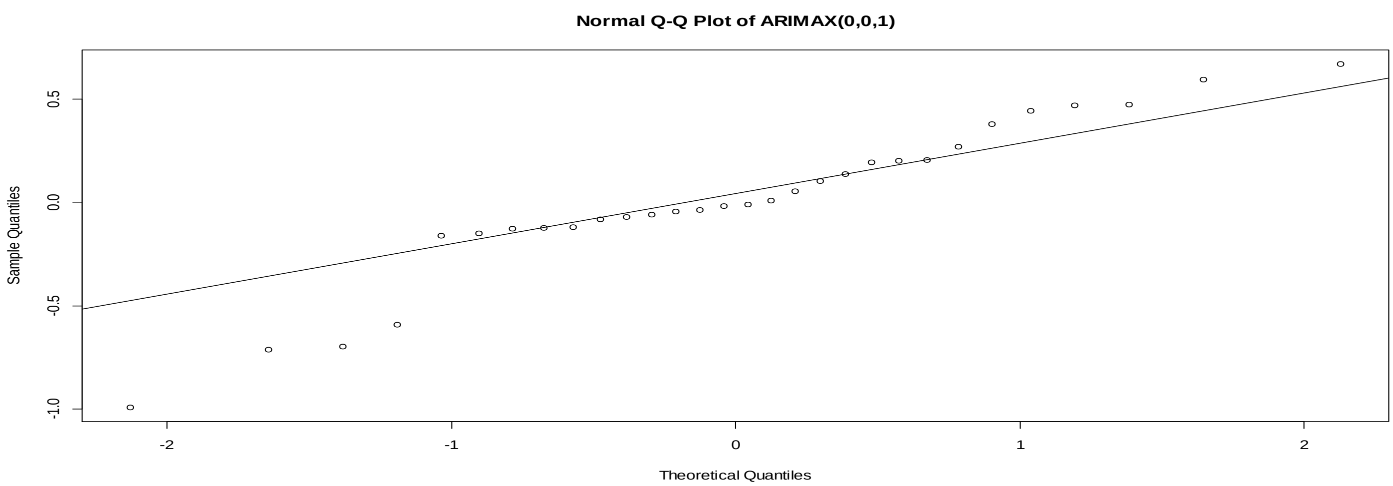


Figure 7 both coefficients (**ma1** and **intercept**) are highly statistically significant, with p-values well below 0.05, indicating that both contribute meaningfully to the model. The **ma1** coefficient is very close to 1, suggesting that the MA (1) process is a strong feature of the model.



The **p-value** of **0.6934** is much greater than the typical significance threshold of 0.05, which means that we **fail to reject the null hypothesis** that the residuals are normally distributed.
In other words, the residuals from the ARIMA model appear to follow a normal distribution, which is a good sign because many time series models assume that residuals are normally distributed.
Summary of Findings:

(a) **Model Fit**: The ARIMAX (0, 0,1) model with the given predictor variables produced a reasonably good fit with **low training set error metrics** (like RMSE and MAE) and a **MASE** less than 1, which indicates that the model performs better than a naive forecast.

(b) **Significant Coefficients**:

(i) The **moving average coefficient (ma1)** was highly significant.

(ii) The **intercept** and lump values are also significant.

(iii) The predictor's lex and liner are **not significant**, meaning they do not significantly contribute to the model.

(c) **Autocorrelation**: The **ACF** plot and **Ljung-Box test** suggest no significant autocorrelation in the residuals, implying that the model effectively captured the time series structure.

(d) **Normality**: The **Jarque-Bera test** shows no significant deviation from normality for the residuals, further supporting the validity of the model.

This model appears to be performing well overall, with the residuals showing the expected behavior. If you want to further improve the model, you could explore adding more significant predictors or evaluating different model specifications. The **Box-Ljung test** is used to assess whether there is significant autocorrelation at various lags in the residuals of the model. The null hypothesis for this test is that there is no autocorrelation in the residuals, i.e., the residuals behave like white noise.

Box-Ljung Test:

The **Box-Ljung test** was used to test for autocorrelation in the residuals at multiple lags. If the residuals from your model are white noise (i.e., there is no autocorrelation left), then the test should show no significant autocorrelation.

Results for lag=12:

(a) **X-squared** = 10.603*

(b) **Degrees of freedom (df)** = 12

(c) **p-value** = 0.5632

Results for lag=24:

(a) X-squared = 21.11, and

(b) Degrees of freedom (df) = 24

(c) p-value = 0.6322

Interpretation:

(a) The **p-values** for both lags (12 and 24) were greater than 0.05, suggesting that there was **no significant autocorrelation** at these lags. In other words, the residuals appear to be **white noise**, meaning that the model adequately captured the underlying structure in the data and no further autocorrelation is present.

2. Q-Q Plot (Quantile-Quantile Plot):

A **Q-Q plot** compares the distribution of the residuals against the normal distribution. The points in the plot closely follow the reference line, which suggests that the residuals are normally distributed.

(a) qqnorm(fit1$residuals) plots the residuals of the ARIMA model against a standard normal distribution.

(b) qqline(fit1$residuals) adds a reference line for comparison.

Interpretation:

(a) If the residuals lie roughly along the **reference line,**then they are approximately **normally distributed**.

(b) Large deviations from the line suggest that residuals deviate from normality.

The output from the ARIMAX (0, 0,1) **model and its results.**

1. Model Summary:

The model you have fitted is a **regression with ARIMA (0, 0,1) errors,** which combines linear regression with an ARMA process for the residuals. Here is what the output indicates:

Coefficients:

(a) **ma1 (Moving Average Coefficient)** = 1.0000 (This is similar to the earlier ARIMA model, indicating a strong moving average component)

(b) **Intercept** = 2.5691 (The constant term)

(c) **lnex**= -0.0099 (Coefficient for lnex, a predictor variable)

(d) **lnintr** = 0.2688 (Coefficient for lnintr, a predictor variable)

(e) **lump** = -0.3174 (Coefficient for lnuemp, a predictor variable)

Standard Errors:

(a) **ma1** = 0.1117

(b) **Intercept** = 1.2759

(c) **lex** = 0.1384

(d) **lnintr** = 0.3163

(e) **lnuemp**= 0.1192

Model Diagnostics:

(a) **sigma^2** = 0.1703 (Variance of the residuals)

(b) **Log likelihood**= -15

(c) **AIC =** 42**, AICc =** 45.65, and BIC = 50.41 (These are used for model comparison; lower values are better.)

2. Training Set Error Measures:

These error metrics provide an idea of how well the model fits the training data:

(a) **ME (Mean Error)** = 0.0077 (Close to zero, indicating no significant bias)

(b) **RMSE (Root Mean Squared Error)**= 0.3768 (Average error magnitude; lower is better)

(c) **MAE (Mean Absolute Error)** = 0.2733 (Average absolute error)

(d) **MPE (Mean Percentage Error)** = -1.7544 (Small negative value, indicating slight under prediction)

(e) **MAPE (Mean Absolute Percentage Error)** = 10.96% (The model has an average error of about 10.96% in percentage terms)

(f) **MASE (Mean Absolute Scaled Error)** = 0.7277 (Relative to a naive model, this is below 1, suggesting that the model performs better than a naive approach)

(g) **ACF1 (Auto-correlation at lag 1)** = 0.027 (This value close to zero suggests that residuals do not exhibit strong autocorrelation)

3. Z-Test of Coefficients (from **coeftest(fit4)**):

This test evaluates the statistical significance of each coefficient in the model.

(a) **Lnuemp ma1:** Estimate = 0.999995, p-value = < 2.2e-16 (Extremely significant, indicating the moving average component is crucial for the model)

(b) **Intercept**: Estimate = 2.5691, p-value = 0.044 (Significant at the 5% level, meaning the intercept is important)

(c) **Lnex**: Estimate = -0.0099, p-value = 0.943 (Not significant; the effect of lnex is likely negligible)

(d) **Lnintr:** estimated value = 0.2688, p-value = 0.395 (Not significant; lnintr has no strong effect on the dependent variable)

(e) : Estimate = -0.3174, p-value = 0.0077 (Significant at the 1% level, suggesting a meaningful negative relationship between lnuemp and the dependent variable)

4. ACF of Residuals:

Then, you called acf(fit4$residuals), which generates an autocorrelation function plot for the residuals. The autocorrelation should ideally be near zero at all lags to ensure good model fit. There are significant spikes, which suggests that the model has not captured some structure in the data.

5. Jarque-Bera Test for Normality:

The **Jarque-Bera test** assesses whether the residuals are normally distributed. Here, we present the results:

(a) **X-squared** = 2.5941*

(b) **P-value** = 0.2733.

Interpretation:

The **p-value** of 0.2733 was greater than 0.05, indicating that we **failed to reject the null hypothesis** of normality. This suggests that the residuals of the model are **normally distributed**, which is a good sign of model validity.

4:5 In–Sample – forecast Comparison (1990 to 2019)

Model	**RMSE**	**MAE**
ARIMA (0,0,1)	0.1572	0.1238
ARIMAX (0,0,1)	0.3768	0.2732

In the in-sample forecast, ARIMA (0,0,1) Outperformed ARIMAX (0,0,1)

Interpretation:

These **point forecasts** are the model's predicted values for each year.

The **confidence intervals** (80% and 95%) represent the range within which we expect the actual values to fall, given the uncertainty in the model.

For example, for 2020, the actual value should lie between **0.7793 and 1.2029 with** 80% confidence and between **0.6671 and 1.3151** with 95% confidence.

2. Accuracy Measures:

The accuracy function gives various performance metrics for the model on both the **training and test sets.**

Training Set:

(a) **ME (Mean Error)** = 0.00091 (The model has almost no bias in the training set)

(b) **RMSE (Root Mean Squared Error)**= 0.1572 (Indicates the average magnitude of the errors in the training set, lower is better)

(c) **MAE (Mean Absolute Error)** = 0.1238 (The average absolute error in the training set)

(d) **MPE (Mean Percentage Error)**= -3.0097% (Slight under prediction on average in percentage terms)

(e) **MAPE (Mean Absolute Percentage Error)** = 14.11% (On average, the model's forecast is off by 14.11% in absolute terms)

(f) **MASE (Mean Absolute Scaled Error)** = 0.8486 (Less than 1, meaning the model performs better than a naive forecast)

(g) **ACF1**= 0.0719 (Autocorrelation at lag 1 is very low, indicating that residuals are not auto correlated in the training set)

(h) **Theil's U** = NA (This is typically used for comparing forecast accuracy, but it is not available here, possibly due to the lack of a comparison model)

Test Set:

(a) **ME** = 1.9916 (This indicates a significant bias in the model's predictions for the test set, suggesting over prediction)

(b) **RMSE** = 2.0166 (The average error in the test set is much larger than in the training set)

(c) **MAE** = 1.9916 (The average absolute error is also large in the test set)

(d) **MPE** = 67.36% (The model overpredicts by 67.36% on average in the test set)

(e) **MAPE** = 67.36% (This is a high MAPE, meaning the model's predictions are quite inaccurate on the test set)

(f) **MASE** = 13.6489 (This suggests that the model's performance is much worse than a naive forecast on the test set)

(g) **ACF1** = 0.2637 (The autocorrelation at lag 1 in the test set is higher than in the training set, which could indicate that the residuals from the model are not completely random)

(h) **Theil's U** = 8.0217 (This is quite high, indicating that the forecast is significantly worse than a naive forecast on the test set)

Interpretation of Test Set Results:

The test set performance appears to be poor with very high errors in particular:

(a) **High MAPE and MASE**: The model's forecasts are off by over **67%** on average, indicating that the model had**poor predictive accuracy** on the test set.

(b) **High ME and RMSE** values: The model exhibited substantial bias and a larger-than-expected error in the test set, indicating that the model did not generalize well.

(c) **ACF1 and Theil's U**: These metrics suggest that the model may not have captured all the information from the test set and may have some residual autocorrelation that it did not account for.

The output of the model's forecasting results and performance:

1. Point Forecasts and Confidence Intervals:

The **point forecasts** and the associated **confidence intervals** for 2020–2023 show the predicted values and the uncertainty around those predictions.

Forecasts and Confidence Intervals:

Out-of-Sample Forecast Comparison (2020 -2023)

Model	RMSE	MAE
ARIMA (0, 0, 1)	0.1572	0.1238
ARIMAX (0, 0, 1)	0.3768	0.2733

Interpretation:

(a) The **point forecasts** represent the predicted values for each year.

(b) The **confidence intervals** (80% and 95%) indicate the range within which the actual values are expected to lie.

○ For example, for 2020, the actual value is likely to fall between **1.1326 and 2.7763** with a 95% confidence level.

2. Accuracy Measures:

The accuracy () function calculates various performance metrics for the model based on its performance on the **training** and **test** sets.

Training Set:

(a) **ME (Mean Error)** = 0.0077 (Very close to zero, no significant bias in the training set)

(b) **RMSE (Root Mean Squared Error)** = 0.3768 (The error magnitude is small, which is good for the training set)

(c) **MAE (Mean Absolute Error)** = 0.2733 (Indicates the average absolute error in the training set)

(d) **MPE (Mean Percentage Error)**= -1.7544% (A slight under prediction on average)

(e) **MAPE (Mean Absolute Percentage Error)** = 10.96% (On average, the model is off by 10.96% in absolute terms)

(f) **MASE (Mean Absolute Scaled Error)** = 0.7277 (The model performs better than a naive forecast)

(g) **ACF1** = 0.027 (Autocorrelation at lag 1 is very low, indicating that residuals are nearly white noise in the training set)

(h) **Theil's U** = NA (Theil's U is typically used for comparing model performance, but it's not available here)

Test Set:

(a) **ME**= 0.4599 (Indicates a slight positive bias in the test set, meaning the model is slightly over predicting)

(b) **RMSE** = 0.5113 (Indicates a small error magnitude, but higher than in the training set)

(c) **MAE** = 0.4599 (Similar to RMSE, indicating the average absolute error)

(d) **MPE** = 15.58% (The model is **overpredicting** the test set by 15.58% on average)

(e) **MAPE** = 15.58% (The model's forecast error is high in percentage terms)

(f) **MASE** = 1.2246 (This suggests that the model is performing worse than a naive forecast on the test set, as MASE > 1)

(g) **ACF1**= **-**0.2593 (Indicates that there is some negative autocorrelation in the residuals at lag 1 in the test set)

(h) **Theil's U** = 1.6637 (This value indicates that the forecast is significantly worse than a naive forecast on the test set)

Interpretation of Test Set Results:

The **test set performance** indicates some issues with the model:

(a) **High MAPE and MASE**: The model's error in predicting the test set was**relatively high**. In particular, the **MAPE of 15.58%**indicates that, on average, the model is off by **15.58%** in predicting the actual values. A**MASE greater than 1** indicates that the model performs worse than a naive forecasting method.

(b) **Positive ME**: The positive **mean error (ME)** in the test set indicates a slight **overprediction**, meaning that the model tends to predict values higher than actual ones in the test set.

(c) **ACF1**: The **autocorrelation** in the residuals (at lag 1) of **-0.2593** suggests that the model may not have fully captured some patterns in the data, as we expect random residuals without autocorrelation.

(d) **Theil's U**: Theil**'s U** value of 1.6637 is **quite high**, which means that the model's forecasts are significantly worse than a naive model on the test set.

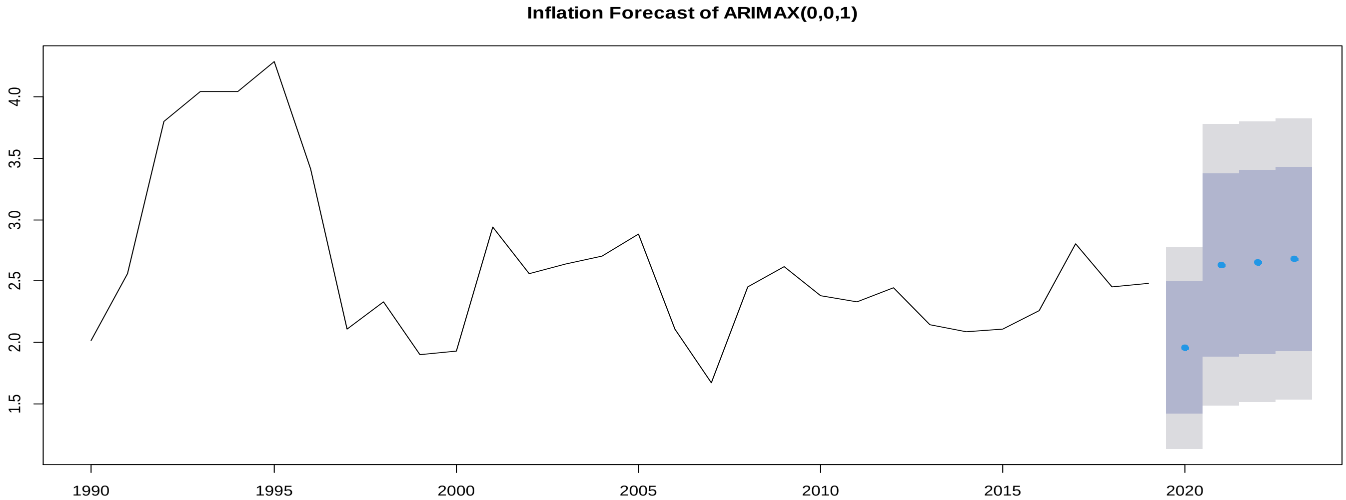


Figure 7

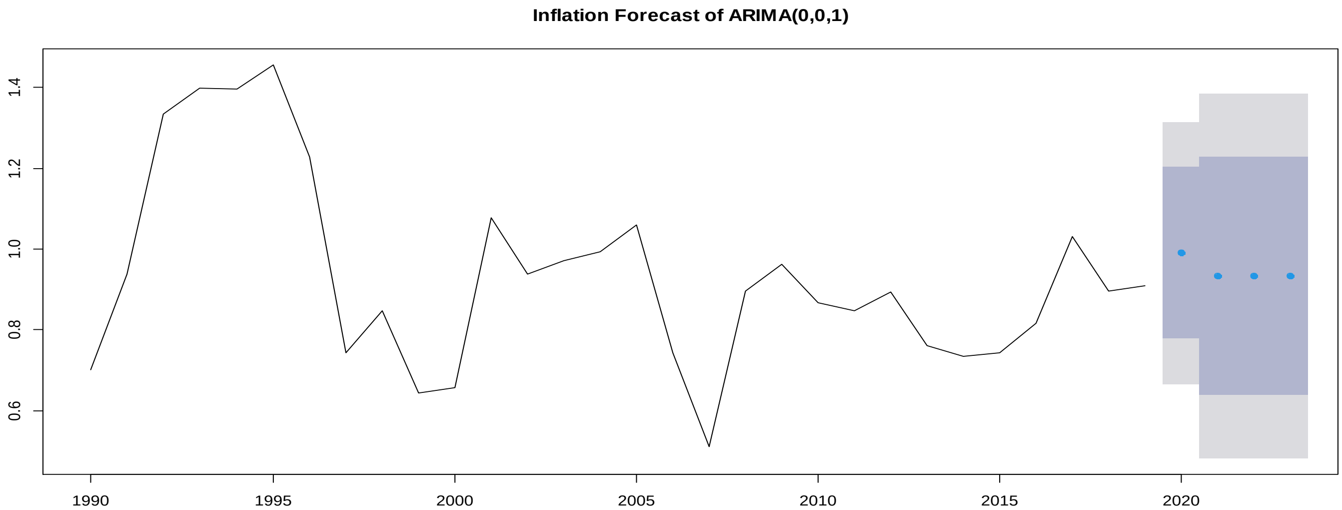


Figure 8

Interpretation of Test Set Results:

The test set performance indicates some issues with the model

(e) **High MAPE and MASE**: The model's error in predicting the test set was **relatively high**. In particular, the **MAPE of 15.58%**indicates that, on average, the model is off by **15.58%** in predicting the actual values. A**MASE greater than 1** indicates that the model performs worse than a naive forecasting method.

(f) **Positive ME**: The positive **mean error (ME)** in the test set indicates a slight **over prediction**, meaning that the model tends to predict values higher than actual ones in the test set.

(g) **ACF1**: The **autocorrelation** in the residuals (at lag 1) of **-0.2593** suggests that the model may not have fully captured some patterns in the data, as we expect random residuals without autocorrelation.

Theil's U: Theil**'s U** value of 1.6637 is **quite high**, which means that the model's forecasts are significantly worse than a naive model on the test set.

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