Economics and Statistics Research Journal (ESRJ)
Volume.16, Number 7; July-2025;
ISSN: 2836-8398 | Impact Factor: 9.97
https://zapjournals.com/Journals/index.php/esrj
Published By: Zendo Academic Publishing

MULTIVARIATE TIME SERIES MODELING OF THE DOLLAR-TO-NAIRA EXCHANGE RATES ON SOME SELECTED ECONOMIC VARIABLES

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Article Info
Keywords: Cointegration, Economic variables, Exchange rate, Granger-causality, VECM
DOI
10.5281/zenodo.16845984

Abstract
This study formulated a multivariate time series model for the monthly USD-to-Naira exchange rate using the time-series VECM approach with stationarity, co-integration, and Granger causality tests. The economic variables used for this study include inflation rate, money supply, and crude oil price, all of which were obtained from the official CBN website from January 2002 to August 2023. The collected data were analysed using VECM after testing for stationarity and confirming the existence of co-integration in the economic variable using E-Views 9.0. The results showed that VECM (1, 1) was the most suitable model for forecasting the dollar-to-naira exchange rate in Nigeria because there was at most one co-integrating vector in the model. The formulation of the VECM model led to the successful comparison of the Naira-USD exchange rate with the other models. Therefore, the study concluded that the vector error correction model (1, 1) was the appropriate model and made recommendations to policymakers to continue monitoring trends in the dollar-to-naira exchange rate, inflation rate, crude oil price, and money supply to implement and sustain policies aimed at maintaining macroeconomic stability.

1.0 Introduction

The recent floating exchange rates in Nigeria and the significant impact of the dollar-to-naira exchange rate on the economy and the living standard of citizens have become a great concern for all Nigerians. The sudden geometric increase in the price of goods and commodities has caused many traders and business tycoons to experience declines and agitation from customers in response to the current price tag on goods (Okafor, 2023). The exchange rate is an important factor in a nation's foreign commerce (Nick, 2022).

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Furthermore, a country's exchange rate serves as a proxy for its degree of global competitiveness in trade. When a country uses gold or another accepted standard, there is a set exchange rate, and each currency is worth a certain amount of the metal or other standard. A speculative activity or the force of supply and demand for exchange rates (conversion units) determine the float (McDonald, 2023). Therefore, it has become imperative for policymakers, companies, and concerned individuals to critically understand exchange rate fluctuations and how they interact with other economic variables (Odukoya & Adio, 2022).
Hence, this study adopted a multivariate time series modeling (MTSM) which help provide insights into the intricate interactions between economic variables that influence the exchange rate. The specific objectives include identifying the dollar-to-naira exchange rate trend, inflation rate, crude oil price, and money supply, exploring causal relationships between different economic variables using Granger causality tests, fitting an appropriate multivariate time series model to the economic variables, and forecasting the future exchange rate using the model formulated

2.0 Methodology

Stationarity testing using unit root tests

The Dickey–Fuller test is a statistical test used to check for stationarity in time series. This is a type of unit root test, through which we determine whether the time series has any unit root (Santra, 2023). The augmented Dickey–Fuller test assumes an AR(p)-type time-series model and is mathematically represented as follows:

$$y_t = \mu + \sum_{i=1}^{p} \varphi_i y_{t-1} + \varepsilon_t \tag{1}$$

The time series is considered stationary when the differenced and null hypothesis is rejected.

Autoregressive model

Autoregressive models operate under the premise that past values have an effect on current values, which makes the statistical technique popular for analyzing nature, economics, and other processes that vary over time. In an AR(P) autoregressive process, the current value is based on the P-lagged immediately preceding value (Mehandzhiyski, 2023). An AR(p) is given as follows:

$$y_t = c + \sum_{i=1}^{p} \varphi_i y_{t-1} + \varepsilon_t \tag{2}$$

This model is similar to a multiple regression with lagged Y_t as predictors.

Vector autoregressive (VAR) model

VAR models are used for multivariate time series. Each variable is a linear function of past lags of itself and past lags of the other variables (Penn State College, 2020).

$$\begin{aligned} y_{t,1} &= c_1 + \sum_{i=1}^{p} \varphi_{i1} y_{t-1,1} + \varepsilon_{t,1} \\ y_{t,2} &= c_2 + \sum_{i=1}^{p} \varphi_{i2} y_{t-1,2} + \varepsilon_{t,2} \\ &\vdots \\ y_{t,n} &= c_n + \sum_{i=1}^{p} \varphi_{in} y_{t-1,n} + \varepsilon_{t,n} \end{aligned} \tag{3}$$

Each variable is a linear function of the lag 1 values for all variables in the set.

Assumptions of the VAR Model

The linearity assumptions assume a linear relationship between the variables and lags. The stationarity assumption implies that the variables' statistical properties, such as mean and variance, do not change over time. The assumption of no endogeneity implies that each variable is treated as exogenous, and its past values contribute to the current value. The assumption of no heteroscedasticity implies a constant variance of the residuals over time.

No serial correlation in residuals implies that the residuals should not exhibit serial correlation. Finally, the normality of residuals (for inference) assumes that the residuals are normally distributed

The economic data used in the study are the monthly exchange rate of the dollar to naira, money supply (M2), inflation rate, and crude oil price from January 2006 to October 2023. These data were obtained from the Official CBN website and https://www.exchangerates.org.uk/USD-NGN-exchange-rate-history.html. The study adopted the Box–Jenkins approach under the following stages: model identification, estimation, diagnostics, selection, and forecasting. The analyses were performed using E-Views 9.0.

3.0 Results

The results of the study using the techniques discussed above are shown in the graphs and tables below. The time plots of the log transformed variables are presented in Fig. 1 below.

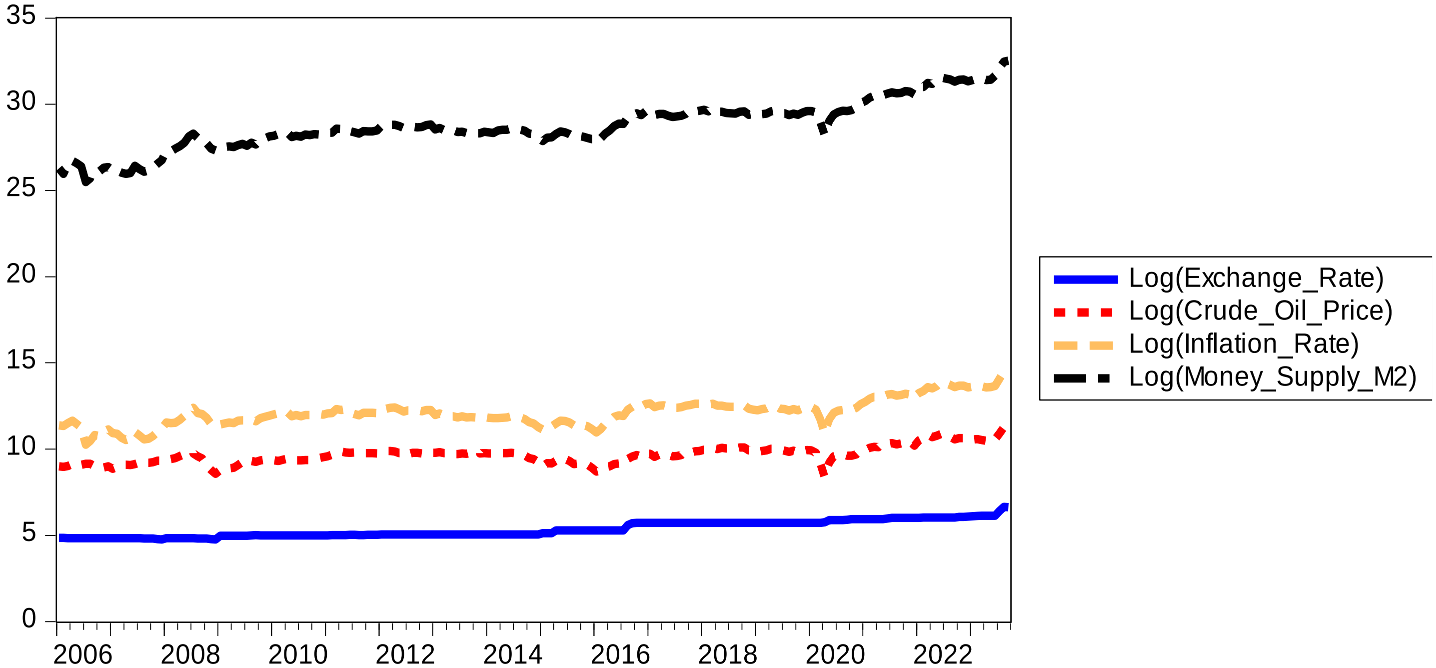

Fig1: Time plot of the log economic variables from 2006 -2023

Table 1: Descriptive statistics of the log economic variables (2006- 2023)

Statistic	**Log(Crude_Oil_Price)**	**Log(Exchange_Rate)**	**Log(Inflation_Rate)**	**Log(Money_Supply_M2)**
Mean	4.31	5.35	2.46	16.64
Median	4.32	5.05	2.48	16.73
Maximum	4.93	6.64	3.28	18.01
Minimum	2.66	4.75	1.10	14.62
Std. Dev.	0.36	0.46	0.37	0.72
Skewness	-0.68	0.51	-0.66	-0.36
Kurtosis	4.12	1.96	3.95	2.70
Jarque-Bera	27.69	18.79	23.59	5.40
Probability	< 0.001	< 0.001	< 0.001	0.067
Observations	213	213	213	213

Table 1 shows that the log exchange rate ranges from 4.752 to 6.646, with a mean of 5.35 (SD). = 0.46) in 2006 to 2023. Hence, there is a broad spectrum of exchange rate values, indicating potential currency variability and fluctuations. The log inflation rates ranged from 1.099 to 3.285, with a mean of 2.46 (std dev. = 0.37). This signifies diverse inflationary trends, with the mean offering a central tendency around which the logarithmic inflation values revolve.

Furthermore, the Log crude oil prices ranged from 2.659 to 4.933, with a mean of 4.31 (SD. = 0.36). This reveals a logarithmic view of the crude oil market, showcasing the variability in oil prices over the observed period. Lastly, the log money supply fluctuated from 14.62 to 18.01, with a mean of 16.64 (SD. = 0.72). The table also shows that the economic variables were not normally distributed.

Table 2 ADF test of the different time series

Variables	**Dickey Fuller (alpha)**	**p-value**
Ln (Exchange Rate)	-5.1507	0.01
Ln (inflation Rate)	-9.3464	0.01
Ln (Crude Oil Price)	-6.3866	0.01
Ln (Money Supply)	-6.1614	0.01

Table 2 shows that there is sufficient evidence to conclude the presence of a unit root in the time series data ($p < 0.05$). Due to the presence of a unit root, which indicates that the time series data are not stationary, further transformation of the non-stationary data to a stationary data by differencing the data is required. Therefore, the variables were differenced at lag 1 to attain stationarity for appropriateness and usefulness in modeling.

Table 3: Determination of the appropriate lags for the proposed model

Lag Order	AIC(n)	HQ(n)	SC(n)	FPE(n)
1	-21.68723*	-21.55701	-21.36523	3.81E-10
2	-21.64249	-21.4081	-21.06288	3.99E-10
3	-21.58195	-21.24339	-20.74475	4.24E-10
4	-21.61683	-21.1741	-20.52203	4.10E-10
5	-21.64345	-21.09655	-20.29104	4.00E-10
Best Lag*	1	1	1	1

Table 3 reveals the lag order selection for the VAR model. Various criteria, including Akaike information criterion (AIC), Hannan-Quinn Information Criterion (HQIC), Schwarz information criterion (SIC), and final prediction error (FPE), were employed to assess different lag lengths (n). The results indicate that the lag-1 model consistently yields the lowest values across all criteria. Therefore, the optimal lag order for the VAR model is defined to be 1. The time plot of the lag 1 differenced log transformed variables is given as follows:

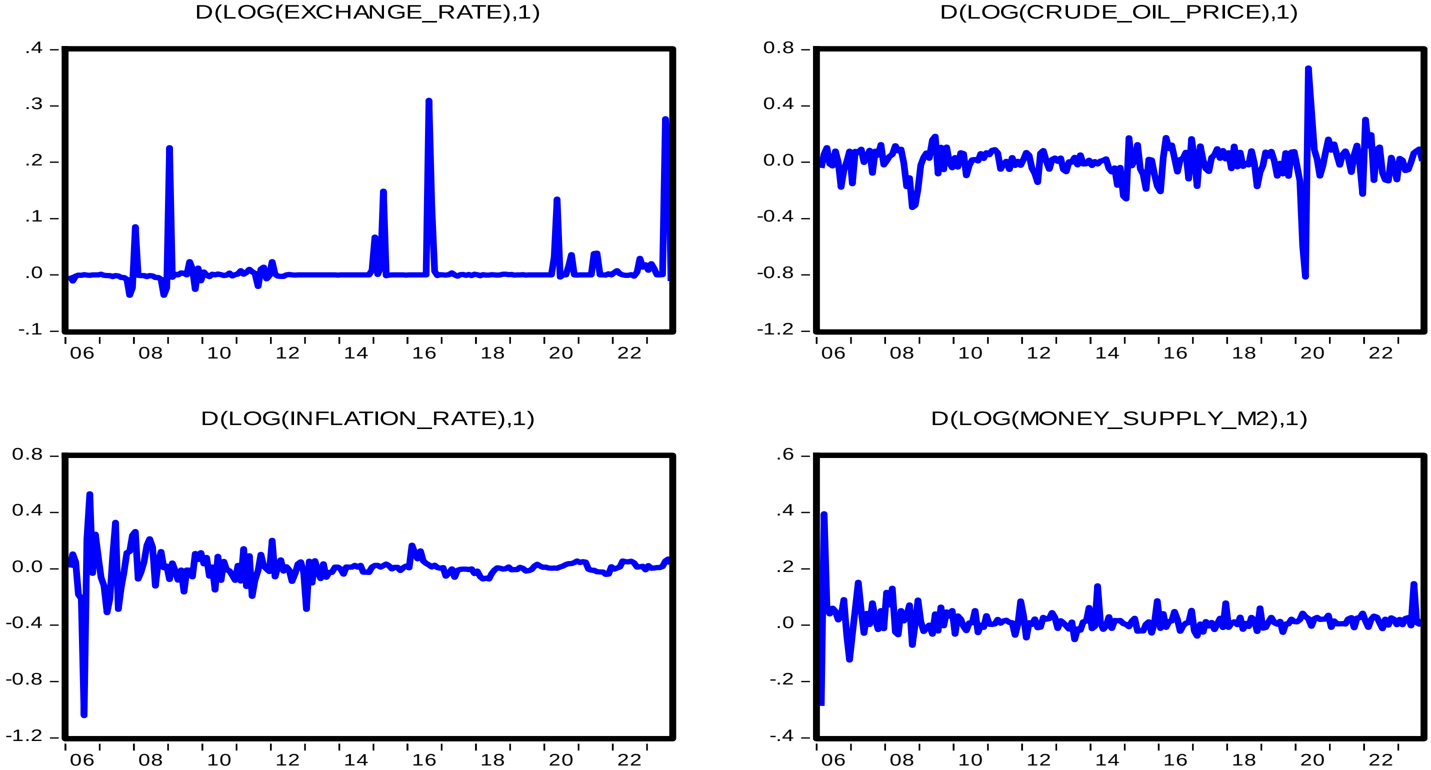

Fig 2: Time plot of the different economic data at order 1

Table 4: Vector autoregressive lag 1 model

Error Correction:	D(log(Crude_Oil_Price),2)	D(log(Exchange_Rate),2)	D(inflation_Rate,2)	D(log(Money_Supply_M2),2)
CointEq1	**-0.3718**	**-0.0822**	-0.0824	**0.1357**
	(0.0750)	(0.0225)	(0.5561)	(0.0198)
	[-4.9551]	[-3.6515]	[-0.1482]	[6.8514]
D(log(Crude_Oil_Price(-1)),2)	**-0.1838**	0.0107	0.0326	**-0.1283**
	(0.0794)	(0.0238)	(0.5881)	(0.0209)
	[-2.3157]	[0.4482]	[0.0554]	[-6.1265]
D(log(Crude_Oil_Price(-2)),2)	**-0.1610**	0.0256	0.3238	**-0.0634**
	(0.0728)	(0.0218)	(0.5394)	(0.0192)
	[-2.2118]	[1.1715]	[0.6003]	[-3.3017]
D(log(Exchange_Rate(-1)),2)	0.4035	**-0.4273**	0.7962	-0.0917
	(0.2260)	(0.0678)	(1.6743)	(0.0596)
	[1.7858]	[-6.3026]	[0.4756]	[-1.5382]
D(log(Exchange_Rate(-2)),2)	0.2426	**-0.4096**	-0.2952	-0.0049
	(0.2248)	(0.0674)	(1.6656)	(0.0593)
	[1.0795]	[-6.0726]	[-0.1772]	[-0.0826]
D(Inflation_Rate(-1),2)	-0.0065	-0.0010	**-0.5600**	-0.0007
	(0.0095)	(0.0028)	(0.0701)	(0.0025)
	[-0.6906]	[-0.3695]	[-7.9934]	[-0.2826]
D(Inflation_Rate(-2),2)	-0.0049	-0.0027	**-0.1614**	0.0046
	(0.0095)	(0.0028)	(0.0704)	(0.0025)
	[-0.5123]	[-0.9445]	[-2.2939]	[1.8319]
	-1.1313	-0.0634	-0.6653	-0.1318

D(log(Money_Supply_M2(-1)),2)	(0.3055)	(0.0917)	(2.2639)	(0.0806)
	[-3.7028]	[-0.6922]	[-0.2939]	[-1.6346]
D(log(Money_Supply_M2(-2)),2)	**-0.5581**	0.0121	-1.5662	-0.0660
	(0.1951)	(0.0585)	(1.4453)	(0.0515)
	[-2.8613]	[0.2066]	[-1.0836]	[-1.2826]
C	-0.0024	0.0009	0.0041	-0.0002
	(0.0096)	(0.0029)	(0.0710)	(0.0025)
	[-0.2513]	[0.3039]	[0.0573]	[-0.0938]
R-squared	0.2861	0.3340	0.2607	0.4355
Adj. R-squared	0.2538	0.3039	0.22731	0.4100
Sum sq. resids	3.8030	0.3424	208.8027	0.2649
S.E. equation	0.1382	0.0415	1.0243	0.0365
F-statistic	8.8621	11.0880	7.7989	17.0619
Log likelihood	122.13	373.72	-296.46	400.53
Akaike AIC	-1.073	-3.481	2.933	-3.737
Schwarz SC	-0.913	-3.321	3.092	-3.577
Mean dependent	-0.0004	-4.23E-05	0.0015	-0.0002
S.D. dependent	0.1600	0.0497	1.1653	0.0475

The VECM models shown in Table 4 reveal that crude oil price is self-adjusting in the long run for the cointegration correcting term for the crude oil prices equation as 37.18% of the disequilibrium will be corrected each month. It also shows that crude oil price is also negatively autocorrelation in the first and second lag in the short run, implying that it was self-reinforcing. Furthermore, it also shows that first (-1.1313) and second (-0.5581) lagged money supply significantly reduces crude oil prices, whereas the exchange rate and inflation rate do not influence crude oil prices.

Table 4 also reveals that the exchange rate is self-adjusting in the long run for the cointegration correcting term for the exchange rate equation as 8.22% of its disequilibrium will be corrected each month. It also shows that the exchange rate is negatively autocorrelation in the first (-0.4273) and second (-0.4096) lag in the short run, implying that it was self-reinforcing. However, none of the other economic variables (crude oil price, inflation rate, and money supply) significantly influence the exchange rate in the short run ($p > 0.05$).

In addition, Table 4 reveals that for the co-integration correcting term for the inflation rate equation, the inflation rate is not self-adjusting eventually because its disequilibrium is not corrected each month. It also shows that the exchange rate is also negatively autocorrelation in the first (-0.5600) and second (-0.1614) lag in the short run, implying that it was also self-reinforcing. However, none of the other economic variables (crude oil price, exchange rate, and money supply) significantly influence the exchange rate in the short run ($p > 0.05$).

Finally, Table 4 reveals that for the cointegration correcting term for the money supply (M2) equation, M2 is self-adjusting in the long run as 13.57% of its disequilibrium will be corrected each month. It also shows that crude oil price negatively influenced money supply in the first (-0.1283) and second (-0.0634) lag in the short run but was not self-reinforcing. Furthermore, it reveals that the exchange rate and inflation rate do not significantly influence money supply in the short run ($p > 0.05$).

The table further shows that the adjusted proportion of variance explained by each variable lagged equation in descending order is money supply (41.0%), followed by exchange rate (30.4%), crude oil equation (25.4%), and inflation rate (22.7%). Similarly, using the Akaike information criteria (AIC), money supply (-3.737) was the better model with the lowest AIC, followed by exchange rate equation (-3.481), crude oil price (-1.073 and lastly inflation rate (2.933). The standard error of the equations is also ordered as follows: money supply (0.0365), exchange rate (0.0415), crude oil equation (0.1382), and inflation rate (1.0243). By these three statistics, the money supply equation is rated the best, whereas the inflation rate equation is the least rated.

Diagnostic assumption test for the models

Serial correlation, normality, and heteroskedasticity tests

These tests whether the residuals of the model exhibit correlation at different lags using the Portmanteau test (asymptotic) for serial correlation, the normality of the residuals using the Jarque Bera test, and the homoscedasticity (constant variance) using the ARCH test.

Table 5: Portmanteau test (asymptotic) for serial correlation, Jarque-Bera test of normality of the residuals, and the ARCH test

The Portmanteau Test (asymptotic)	Chi Square	234.86
	df	228
	p-value	0.3634
Jarque-Bera test	Chi Square	18295
	df	8
	p-value	2.2e-16
Autoregressive Conditional Heteroskedasticity (ARCH) test	Chi Square	313.17
	df	100
	p-value	2.2e-16

The null hypothesis of no serial correlation in the residuals of the time series model is not rejected ($p > 0.05$) (Table 5). Hence, no serial correlation exists in the residuals of the model. This means that the residuals (the differences between the observed values and the values predicted by the model) do not exhibit a systematic pattern of correlation with their own lagged values. In other words, the residuals are independent over time. Hence, the model adequately explained the systematic patterns, leaving only random noise in the residuals.

Table also shows that the Jarque-Bera test obtained a chi-square value of 243.86 associated with a p-value < 0.05. The null hypothesis of residual normality was rejected ($p < 0.05$). Hence, the model's residuals are not normally distributed (($p < 0.05$). Therefore, the model should be used with caution.

Finally, the table shows that the ARCH test obtained a chi-square value of 313.17 associated with a p-value < 0.05. The null hypothesis of homoscedasticity, i.e., the constant variance of the errors, is rejected ($p < 0.05$). Hence, the model exhibits heteroskedasticity, which further implies that the variance of the errors is not constant, and the model must be used with care.

Granger causality test

Table 6: Granger causality test results

Null Hypothesis	p-value	[a]Remarks
The inflation rate does not Granger-cause the exchange rate.	0.036	reject H_0
Crude oil price does not Granger-cause exchange rate.	0.003	reject H_0
Money supply does not Granger-cause the exchange rate	0.044	reject H_0

[a]Decisions at the 5% level of significance

Table 6 shows the Granger causality tests, which reject the null hypothesis. Hence, the one-period lagged values of the inflation rate, crude oil price, and money supply contain information useful for predicting changes in the exchange rate.

4.0 Discussions

The findings of this research extend those of Alagidede and Muadu (2016), who explored the causes of real exchange rate volatility and its impact on Ghana's economic growth. While Alagidede and Muadu (2016) identified various factors influencing exchange rate fluctuations in Ghana, this study specifically focuses on the Nigerian economy. However, despite the geographical difference, both studies emphasised the importance of factors such as growth in government expenditure, money supply, and terms of trade shocks in shaping exchange rate volatility.

This study's exploration of the relationship between inflation rate and the dollar-to-naira exchange rate volatility in Nigeria shares similarities with the findings of Fadii et al. (2011) in the Malaysian economy. Both studies employ a time-series VECM approach, incorporating stationarity, co-integration, stability, and Granger causality tests. Fadii et al. (2011) found that the inflation rate impacts the interest rate, subsequently influencing the exchange rate, aligning with the results of this study where the inflation rate and money supply are significant predictors in the VECM model.

Olaniran (2019) modeled the short-run relationship between the Naira-USD exchange rate and crude oil price in Nigeria. This study explores a broader set of variables and adopts a VECM for forecasting. While Olaniran (2019) did not find evidence of co-integration between the variables because he considered just two variables in his study, this study considered four economic variables and reveals that there is at most one co-integration relationship between inflation rate, crude oil price, money supply, and exchange rate.

The results of this study align with the insights presented by Azubuike and Kosemoni (2017), who conducted a comparative study to determine the most effective forecasting model for Average Monthly Exchange Rates (AMER). While Azubuike and Kosemoni (2017) found that the univariate time series model fitted better for certain currencies, such as the dollar, pounds sterling, yen, WAUA, and CFA, this research focuses on a specific currency, the Naira-USD exchange rate. Thus, this study extends their findings by employing the VECM for forecasting, emphasizing the relevance of this model for the USD exchange rate.

5.0 Conclusions

The variables attained stationarity after the first differencing of the dataset, as confirmed by the ADF test for stationarity. Granger causality tests revealed the significant influence of past inflation rate, crude oil price, and money supply on predicting exchange rate changes. The selection of the optimal lag order for the VAR model was guided by the Akaike information criterion, which further established the statistical relevance of the formulated multivariate time series model. However, the fitted model violated the assumptions of normality and homoscedasticity, implying that the model must be used with utmost care. Further studies can be conducted using appropriate methods to transform the data to conform to the assumptions of the linear model.

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